CHAPTER

14 Geometry: Measuring Area and Volume

What does architecture have to do with math?

Two- and three-dimensional figures are often found in architecture. The Rock and Roll Hall of Fame in Cleveland, Ohio, contains two-dimensional figures such as triangles, rectangles, and parallelograms, and three-dimensional figures such as prisms, pyramids, and cylinders. The properties of geometric figures can be used to find the area and the volume of buildings.

You will solve a problem about architecture in Lesson 14-2.

GETTING STARTED

Diagnose Readiness

Take this quiz to see if you are ready to begin Chapter 14. Refer to the lesson or page number in parentheses for review.

Vocabulary Review

Complete each sentence.

1. A(n) ___?___ is a number expressed using exponents. (Lesson 1-4)
2. The number that is multiplied in a power is called the ___?___. (Lesson 1-4)
3. ___?___ is the distance around a circle. (Lesson 4-6)

Prerequisite Skills

Evaluate each expression. (Lesson 1-4)

4. 8^2
5. $(1.2)^2$
6. $(0.5)^2$
7. 11^2
8. 7^2
9. 10^2

Estimate each sum. (Lesson 3-4)

10. $17.6 + 8.41 + 3.2$
11. $20.9 + 4.25 + 9.1$
12. $2.7 + 6.9 + 13.8$
13. $15.67 + 11.8 + 7.3$

Multiply. (Lesson 7-2)

14. $\frac{1}{2} \times 6 \times 6$
15. $\frac{1}{2} \times 5 \times 8$
16. $\frac{1}{2} \times 8 \times 3$
17. $\frac{1}{2} \times 4 \times 7$

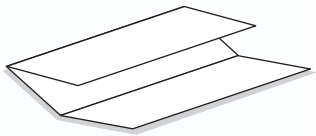
Multiply. (Page 590)

18. $2 \times 7 \times 5$
19. $9 \times 6 \times 4$
20. $4 \times 11 \times 3$
21. $10 \times 8 \times 2$

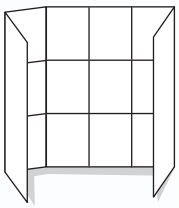
FOLDABLES Study Organizer

Area and Volume Make this Foldable to help you organize information about measuring area and volume.

STEP 1 **Fold**
Fold a sheet of 11" × 17" paper in thirds lengthwise.



STEP 2 **Open and Fold**
Fold a 2" tab along the short side. Then fold the rest into fifths.



STEP 3 **Unfold and Label**
Unfold and draw lines along the folds. Label as shown.

	parallelograms	triangles	circles	3-D figures	rectangular prisms
Area					
Volume					

Noteables™ **Chapter Notes** Each time you find this logo throughout the chapter, use your *Noteables™: Interactive Study Notebook with Foldables™* or your own notebook to take notes. Begin your chapter notes with this Foldable activity.

Math nline **Readiness** To prepare yourself for this chapter with another quiz, visit **msmath1.net/chapter_readiness**

14-1 Area of Parallelograms

What You'll LEARN

Find the areas of parallelograms.

NEW Vocabulary

base
height

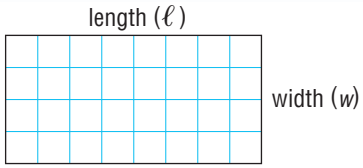
HANDS-ON Mini Lab

Materials
- grid paper
- scissors

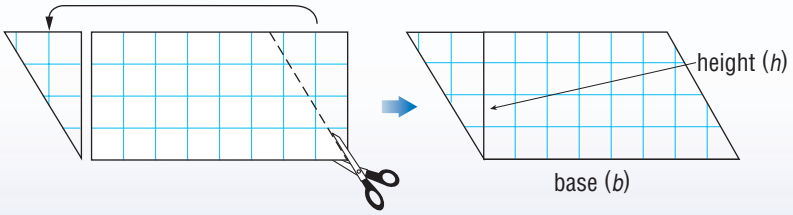
Work with a partner.

You can explore how the areas of parallelograms and rectangles are related.

STEP 1 Draw and then cut out a rectangle as shown.

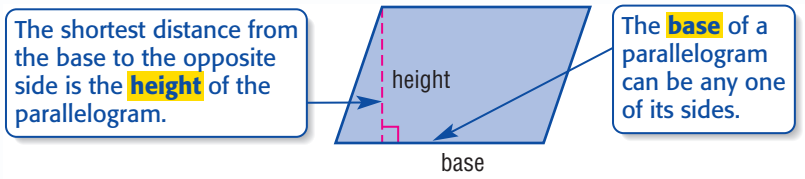


STEP 2 Cut a triangle from one side of the rectangle and move it to the other side to form a parallelogram.



1. How does a parallelogram relate to a rectangle?
2. What part of the parallelogram corresponds to the length of the rectangle?
3. What part corresponds to the rectangle's width?
4. Write a formula for the area of a parallelogram.

In the Mini Lab, you showed that the area of a parallelogram is related to the area of a rectangle. To find the area of a parallelogram, multiply the measures of the base and the height.

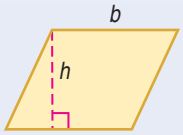


Noteables™ **Key Concept: Area of a Parallelogram**

Words The area A of a parallelogram is the product of any base b and its height h.

Symbols $A = bh$

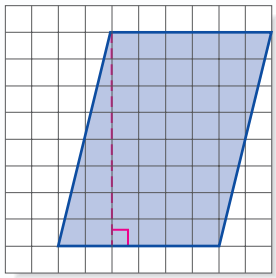
Model



EXAMPLES Find Areas of Parallelograms

Find the area of each parallelogram.

1



The base is 6 units, and the height is 8 units.

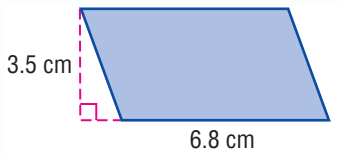
$A = bh$

$A = 6 \cdot 8$

$A = 48$

The area is 48 square units or 48 units^2.

2



$A = bh$

$A = 6.8 \times 3.5$

$A = 23.8$

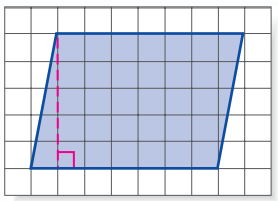
The area is 23.8 square centimeters or 23.8 cm^2.

READING Math

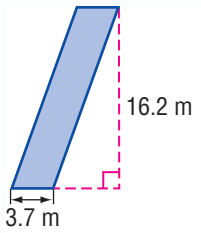
Area Measurement
An area measurement can be written using abbreviations and an exponent of 2.
For example:
square units = units^2
square inches = in^2
square feet = ft^2
square meters = m^2

Your Turn **Find the area of each parallelogram. Round to the nearest tenth if necessary.**

a.



b.



REAL-LIFE CAREERS

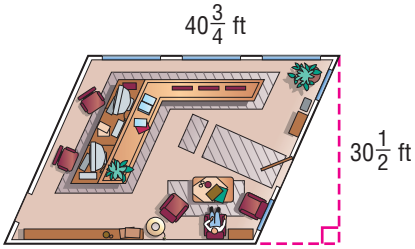
How Does an Architect Use Math?
Architects use geometry when they find the area of buildings.

Research
For information about a career as an architect, visit:
msmath1.net/careers

Many real-life objects are parallelograms.

EXAMPLE Use Area to Solve a Problem

1 **ARCHITECTURE** **An architect is designing a parallelogram-shaped lobby for a small office building. What is the area of the floor plan?**



Since the floor plan of the lobby is a parallelogram, use the formula $A = bh$.

$A = bh$ Area of a parallelogram

$A = \left(40\frac{3}{4}\right)\left(30\frac{1}{2}\right)$ Replace b with $40\frac{3}{4}$ and h with $30\frac{1}{2}$.

$A = \left(\frac{163}{4}\right)\left(\frac{61}{2}\right)$ Estimate $40\frac{3}{4} \times 30\frac{1}{2} \rightarrow 40 \times 30 = 1{,}200$

$A = \frac{9{,}943}{8}$ or $1{,}242\frac{7}{8}$ Write the mixed numbers as improper fractions. Multiply. Then simplify.

The area of the lobby's floor plan is $1{,}242\frac{7}{8}$ square feet.

Notice that this is reasonable compared to the estimate of 1,200.

msmath1.net/extra_examples

Tony Hopewell/Getty Images

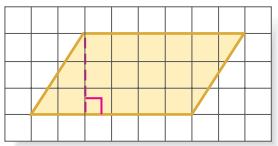
Skill and Concept Check

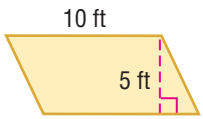
1. Writing Math Explain how the formula for the area of a parallelogram is related to the formula for the area of a rectangle.

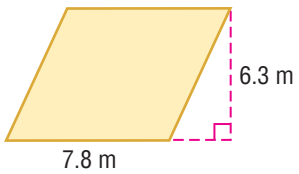
2. **OPEN ENDED** Draw and label two different parallelograms each with an area of 16 square units.

GUIDED PRACTICE

Find the area of each parallelogram. Round to the nearest tenth if necessary.

3. 

4. 

5. 

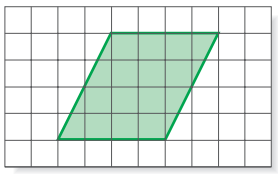
Practice and Applications

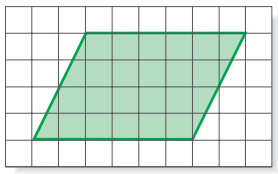
HOMEWORK HELP

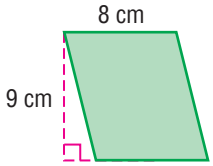
For Exercises	See Examples
6–11	1, 2
12–13, 15, 18	3

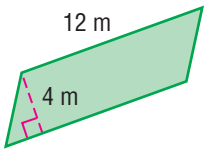
Extra Practice See pages 622, 637.

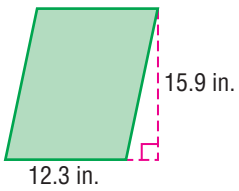
Find the area of each parallelogram. Round to the nearest tenth if necessary.

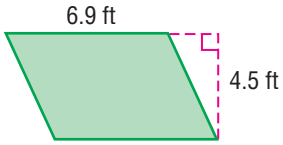
6. 

7. 

8. 

9. 

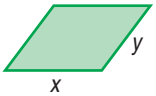
10. 

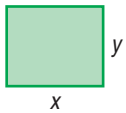
11. 

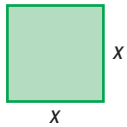
12. What is the measure of the area of a parallelogram whose base is $8\frac{4}{5}$ inches and whose height is $6\frac{3}{8}$ inches?

13. Find the area of a parallelogram with base 6.75 meters and height 4.8 meters.

14. **ALGEBRA** If $x = 5$ and $y < x$, which parallelogram has the greatest area?

a. 

b. 

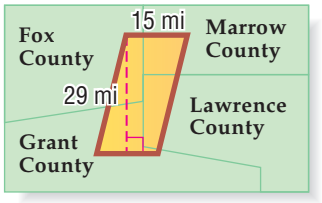
c. 

15. What is a reasonable estimate for the area of a parallelogram with a base of $19\frac{3}{4}$ inches and a height of $15\frac{1}{8}$ inches?

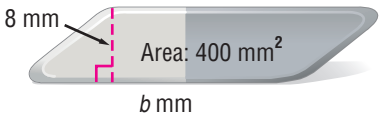
16. **MEASUREMENT** How many square feet are in 4 square yards?

17. **MEASUREMENT** Find the number of square inches in 9 square feet.

18. **WEATHER** A local meteorologist alerted people of a thunderstorm warning for the region shown on the map. What is the area of the region that is under a thunderstorm warning?



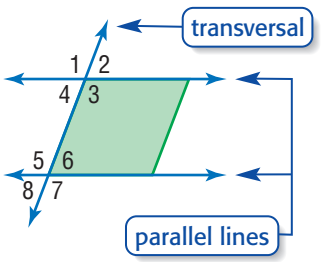
ERASERS **For Exercises 19–20, use the eraser shown.**



19. Write an equation to find the measure of the base of the side of the eraser.
20. Find the measure of the base of the side of the eraser.

21. **CRITICAL THINKING** The base and height of a parallelogram are doubled. How does the area change?

EXTENDING THE LESSON By extending the sides of a parallelogram, special angles are formed. Notice that the line intersecting a pair of parallel lines is called a *transversal*.

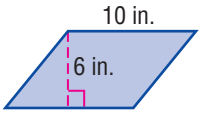


- interior angles: $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$
- exterior angles: $\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$
- alternate interior angles: $\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$
- alternate exterior angles: $\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$
- corresponding angles: $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$

22. Give a definition for each type of angle listed above.
23. Describe the relationship between alternate interior angles.
24. Draw a parallelogram. Then extend its sides. Identify a pair of alternate interior angles and a pair of alternate exterior angles.
25. If $m\angle 3 = 110°$, what angles are congruent to $\angle 3$?
26. If $m\angle 6 = 65°$, find $m\angle 1$.

Spiral Review with Standardized Test Practice

27. **MULTIPLE CHOICE** Find the area of the parallelogram.



(A) 72 in^2 (B) 60 in^2 (C) 30 in^2 (D) 16 in^2

28. **SHORT RESPONSE** What is the height of a parallelogram if its area is 219.6 square meters and its base is 12 meters?

29. Draw a pair of similar quadrilaterals. (Lesson 13-6)

Trace each figure. Then draw all lines of symmetry. (Lesson 13-5)

30. 
31. 
32. 

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Multiply.** (Lesson 7-2)

33. $\frac{1}{2} \cdot 8 \cdot 9$
34. $\frac{1}{2} \cdot 12 \cdot 5$
35. $\frac{1}{2} \cdot 25 \cdot 4$
36. $\frac{1}{2} \cdot 48 \cdot 3$

14-2a HANDS-ON LAB

A Preview of Lesson 14-2

Area of Triangles

In this lab, you will find the area of a triangle using the properties of parallelograms.

What You'll LEARN

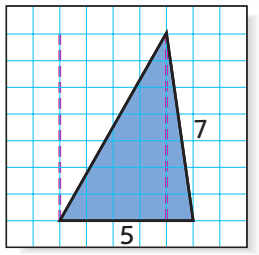
Find the area of a triangle using the properties of parallelograms.

Materials

- grid paper
- colored pencils
- scissors

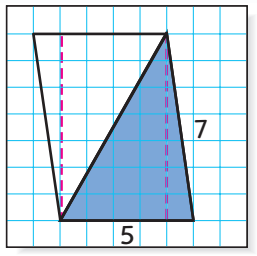
ACTIVITY *Work with a partner.*

STEP 1 Draw a triangle as shown. Label the height and the base.



STEP 2 Draw a dashed line segment that is 7 units high and parallel to the height of the triangle.

STEP 3 Draw a solid line segment that is 5 units long and parallel to the base. Draw another segment to form the parallelogram.

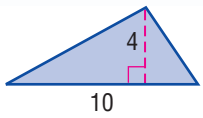


The area of the parallelogram is 5×7 or 35 square units.

The area of the triangle is half the area of the parallelogram. So, the area of the triangle is $35 \div 2$ or 17.5 square units.

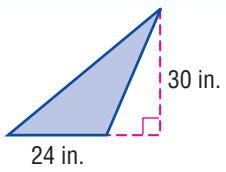
Your Turn

Draw the triangle shown on grid paper. Then draw a parallelogram and find the area of the triangle.



Writing Math

1. Suppose a parallelogram has an area of 84 square units with a height of 7 units. **Describe** a triangle related to this parallelogram, and find the triangle's area, base, and height.
2. **Draw** a parallelogram that is related to the triangle at the right. How could you use the drawing to find the area of the triangle?
3. **Write** a formula for the area of a triangle.



14-2 Area of Triangles

What You'll LEARN

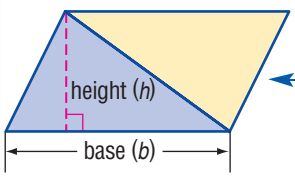
Find the areas of triangles.

WHEN am I ever going to use this?

GAMES Tri-Ominos is a game played with triangular game pieces that are all the same size.

1. Compare the two triangles.
2. What figure is formed by the two triangles?
3. **Make a conjecture** about the relationship that exists between the area of one triangle and the area of the entire figure.

A parallelogram can be formed by two congruent triangles. Since congruent triangles have the same area, the area of a triangle is one half the area of the parallelogram.



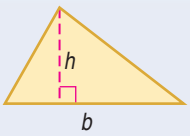
The base of a triangle can be any one of its sides. The height is the shortest distance from a base to the opposite vertex.

Noteables™ **Key Concept: Area of a Triangle**

Words The area A of a triangle is one half the product of the base b and its height h.

Symbols $A = \frac{1}{2}bh$

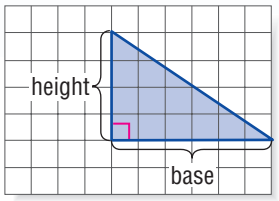
Model



EXAMPLE **Find the Area of a Triangle**

Find the area of the triangle.

By counting, you find that the measure of the base is 6 units and the height is 4 units.



$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(6)(4)$ Replace b with 6 and h with 4.

$A = \frac{1}{2}(24)$ Multiply. $6 \times 4 = 24$

$A = 12$ The area of the triangle is 12 square units.

STUDY TIP

Mental Math You can use mental math to multiply $\frac{1}{2}(6)(4)$. Think: $\frac{1}{2}(6)(4)$ is also $3(4)$ or 12.

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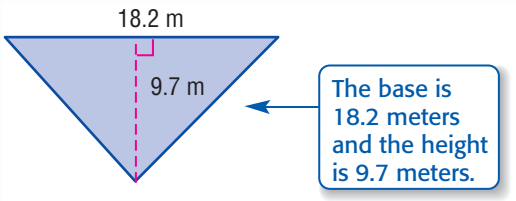
Aaron Haupt

EXAMPLE Find the Area of a Triangle

1 **Find the area of the triangle.**



$A = \frac{1}{2}bh$ Area of a triangle

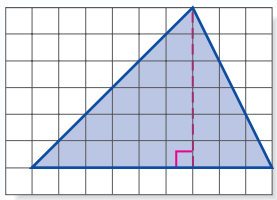
$A = \frac{1}{2}(18.2)(9.7)$ Replace b with 18.2 and h with 9.7.

0.5 × 18.2 × 9.7 ENTER **88.27** Use a calculator.

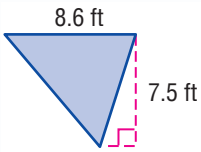
To the nearest tenth, the area of the triangle is 88.3 square meters.

Your Turn **Find the area of each triangle. Round to the nearest tenth if necessary.**

a.



b.

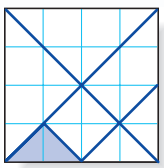


EXAMPLE Use Area to Solve a Problem

Standardized Test Practice

1 **MULTIPLE-CHOICE TEST ITEM**

Which ratio compares the area of the shaded triangle to the area of the large square?



Ⓐ 1 to 4 Ⓑ 1 to 8

Ⓒ 1 to 16 Ⓓ 1 to 32

Read the Test Item

You need to find the ratio that compares the area of the triangle to the area of the large square.

Solve the Test Item

First find the area of the triangle and the area of the square.

Area of Triangle

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(2)(1)$ or 1 unit^2

Area of Square

$A = s^2$

$A = 4^2$ or 16 units^2

Now find the ratio. Since $\frac{\text{area of triangle}}{\text{area of square}} = \frac{1 \text{ unit}^2}{16 \text{ units}^2}$, the ratio is 1 to 16. So, the answer is C.

Test-Taking Tip

Formulas
Most standardized tests list any geometry formulas you will need to solve problems. However, it is always a good idea to familiarize yourself with the formulas before taking the test.

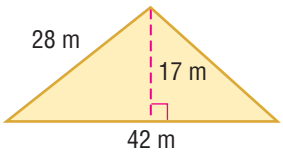
Skill and Concept Check

1. **OPEN ENDED** Draw two different triangles each having an area of 24 square feet.

2. **FIND THE ERROR** Susana and D.J. are finding the area of the triangle. Who is correct? Explain.

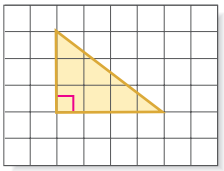
Susana
$A = \frac{1}{2}(28)(42)$
$A = 588\ m^2$

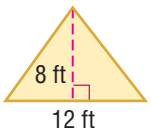
D.J.
$A = \frac{1}{2}(17)(42)$
$A = 357\ m^2$

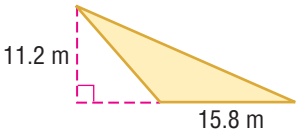


GUIDED PRACTICE

Find the area of each triangle. Round to the nearest tenth if necessary.

3. 

4. 

5. 

6. **SPORTS** The width of a triangular hang glider measures 9 feet, and the height of the wing is 6 feet. How much fabric was used for the wing of the glider?

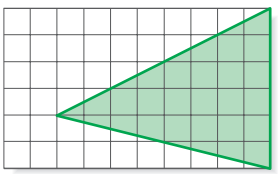
Practice and Applications

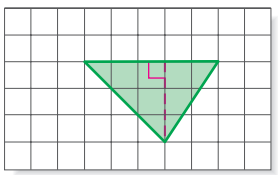
HOMEWORK HELP

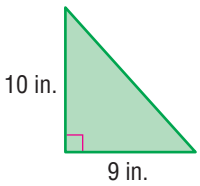
For Exercises	See Examples
7–14, 17–20	1, 2

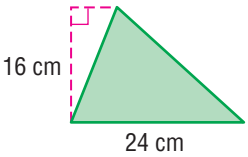
Extra Practice
See pages 622, 637.

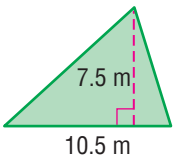
Find the area of each triangle. Round to the nearest tenth if necessary.

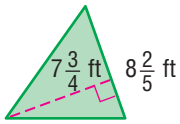
7. 

8. 

9. 

10. 

11. 

12. 

13. height: $4\frac{2}{3}$ in., base: $\frac{3}{4}$ in.

14. height: 7.5 cm, base: 5.6 cm

15. Which is larger, a triangle with an area of 25 square yards or a triangle with an area of 25 square meters?

16. Which is smaller, a triangle with an area of 1 square foot or a triangle with an area of 64 square inches?

17. **ARCHITECTURE** The main entrance of the Rock and Roll Hall of Fame is a triangle with a base of about 241 feet and a height of about 165 feet. Find the area of this triangle.

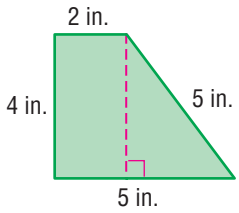
GEOGRAPHY **For Exercises 18 and 19, use the diagram shown and the following information.** The Bermuda Triangle is an imaginary triangle connecting Florida to the Bermuda Islands to Puerto Rico and back to Florida.

18. Estimate the area of the region enclosed by the Bermuda Triangle.

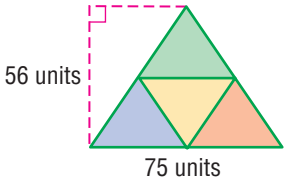
19. Find the actual area of the Bermuda Triangle.

20. **COLLEGE** Jack's dorm room is shaped like a triangle. The college brochure says it has an area of 304 square feet. The room is 15 feet long. Estimate the width of the room at its widest point.

21. Find the area and the perimeter of the figure at the right.



CRITICAL THINKING **For Exercises 22–25, use the figure shown.**



22. Find the area of the figure.

23. Find the measure of the base and height of the four smaller triangles.

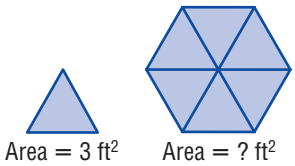
24. What is the area of one small triangle?

25. Is your answer reasonable? Explain.

Spiral Review with Standardized Test Practice

26. **MULTIPLE CHOICE** In the diagram, the triangle on the left has an area of 3 square feet. What is the area of the figure on the right?

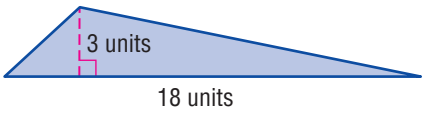
Ⓐ 8 ft^2 Ⓑ 12 ft^2 Ⓒ 18 ft^2 Ⓓ 22 ft^2



27. **MULTIPLE CHOICE** Find the area of the triangle.

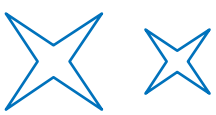
Ⓕ 27 units^2 Ⓖ 36 units^2

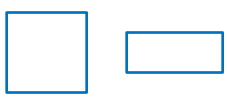
Ⓗ 40 units^2 Ⓘ 54 units^2

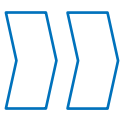


28. **GEOMETRY** Find the area of a parallelogram whose base is 20 millimeters and height is 16 millimeters. (Lesson 14-1)

Tell whether each pair of figures is *similar*, *congruent*, or *neither*. (Lesson 13-6)

29. 

30. 

31. 

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Evaluate each expression.** (Lesson 1-4)

32. 9^2 33. 12^2 34. 0.6^2 35. 1.5^2

14-2b HANDS-ON LAB

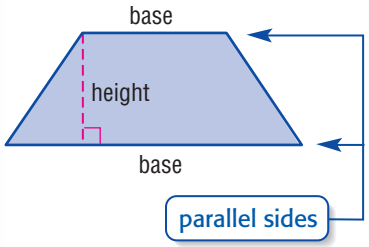
A Follow-Up of Lesson 14-2

Area of Trapezoids

What You'll LEARN

Find the area of a trapezoid using the properties of triangles.

A *trapezoid* is a quadrilateral with one pair of opposite sides parallel. In this lab, you will explore how to find the area of a trapezoid using the formula for the area of a triangle.

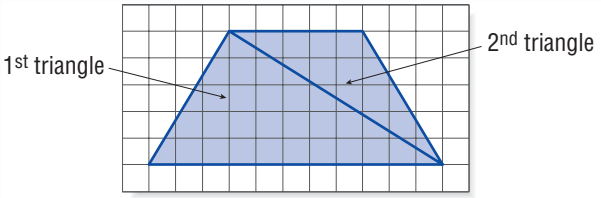


Materials

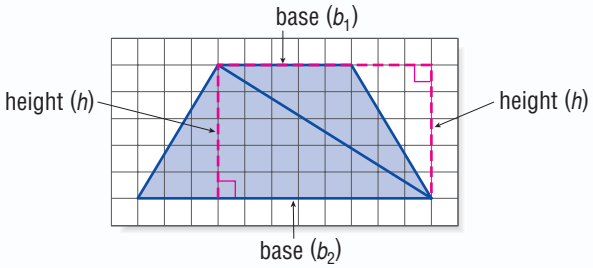
- grid paper

ACTIVITY

STEP 1 Draw a trapezoid. Separate it into two triangles as shown.



STEP 2 Draw and label the height and base of each triangle.



STEP 3 Write a formula for the area of the trapezoid.

$$\text{area of trapezoid} = \text{area of first } \triangle + \text{area of second } \triangle$$
$$= \frac{1}{2}b_1h + \frac{1}{2}b_2h$$
$$= \frac{1}{2}h(b_1 + b_2) \quad \text{Distributive Property}$$

Your Turn

a. Find the area of the trapezoid above.

Writing Math

1. **Explain** why the area of a trapezoid is related to the area of a triangle.
2. Why can $A = \frac{1}{2}b_1h + \frac{1}{2}b_2h$ be written as $A = \frac{1}{2}h(b_1 + b_2)$?
3. **Explain** how you would separate any trapezoid into triangles to find its area.

14-3 Area of Circles

What You'll LEARN

Find the areas of circles.

REVIEW Vocabulary

circumference: the distance around a circle (Lesson 4-6)

MATH Symbols

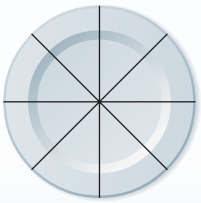
π approximately 3.14

HANDS-ON Mini Lab

Materials
- paper plate
- scissors

Work with a partner.

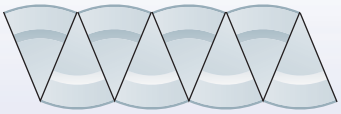
You can use a paper plate to explore the area of circles.



STEP 1 Fold a paper plate into eighths.

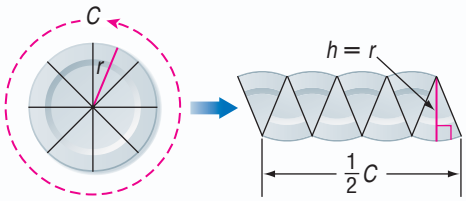
STEP 2 Unfold the plate and cut along the creases.

STEP 3 Arrange the pieces to form the figure shown.



1. What shape does the figure look like?
2. What part of the circle represents the figure's height?
3. Relate the circle's circumference to the base of the figure.
4. How would you find the area of the figure?

A circle can be separated into parts as shown. The parts can then be arranged to form a figure that resembles a parallelogram.



You can use the formula for the area of a parallelogram to find the formula for the area of a circle.

$A = bh$	Area of a parallelogram
$A = \left(\frac{1}{2}C\right)r$	The base is one half the circumference. The height is the radius.
$A = \frac{1}{2}(2\pi r)r$	Replace C with $2\pi r$, the formula for circumference.
$A = \pi \cdot r \cdot r$	Simplify. $\frac{1}{2} \cdot 2 = 1$
$A = \pi r^2$	Simplify. $r \cdot r = r^2$

READING in the Content Area

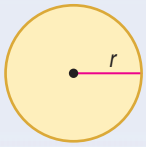
For strategies in reading this lesson, visit msmath1.net/reading.

Noteables™ **Key Concept: Area of a Circle**

Words The area A of a circle is the product of π and the square of the radius r.

Symbols $A = \pi r^2$

Model



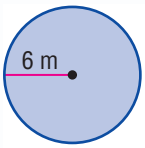
READING Math

Estimation To estimate the area of a circle, you can multiply the square of the radius by 3 since π is about 3.

EXAMPLES Find Areas of Circles

Find the area of each circle to the nearest tenth. Use 3.14 for π.

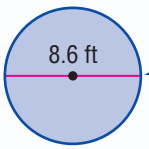
1



$A = \pi r^2$	Area of a circle
$A \approx 3.14 \times 6^2$	Replace π with 3.14 and r with 6. Estimate $3.14 \times 6^2 \rightarrow 3 \times 40 = 120$
$A \approx 3.14 \times 36$	Evaluate 6^2.
$A \approx 113.04$	Use a calculator.

The area is about 113.0 square meters.

2



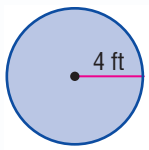
The diameter is 8.6 feet. So, the radius is 8.6 ÷ 2 or 4.3 feet.

$A = \pi r^2$	Area of a circle
$A \approx 3.14 \times 4.3^2$	Replace π with 3.14 and r with 4.3. Estimate $3.14 \times 4.3^2 \rightarrow 3 \times 20 = 60$
$A \approx 3.14 \times 18.49$	Evaluate 4.3^2.
$A \approx 58.0586$	Use a calculator.

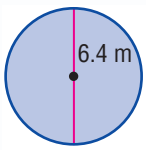
The area is about 58.1 square feet.

Your Turn **Find the area of each circle to the nearest tenth. Use 3.14 for π.**

a.



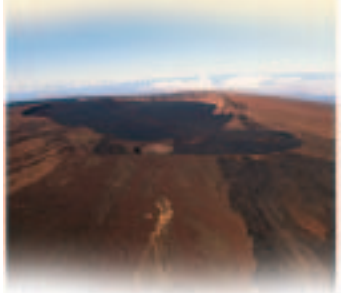
b.



REAL-LIFE MATH

VOLCANOES Shield volcanoes are named for their broad and gently sloping shape that looks like a warrior's shield. In California and Oregon, many shield volcanoes have diameters of three or four miles and heights of 1,500 to 2,000 feet.

Source: U.S. Geological Survey



Many real-life objects are circular.

EXAMPLE Use Area to Solve a Problem

1 **EARTH SCIENCE** **The Belknap shield volcano is located in Oregon. This volcano is circular and has a diameter of 5 miles. About how much land does this volcano cover?**

Use the area formula to find the area of the volcano.

$A = \pi r^2$	Area of a circle
$A \approx 3.14 \times 2.5^2$	Replace π with 3.14 and r with 2.5. Estimate $3.14 \times 2.5^2 \rightarrow 3 \times 6 = 18$
$A \approx 3.14 \times 6.25$	Evaluate 2.5^2.
$A \approx 19.625$	Use a calculator.

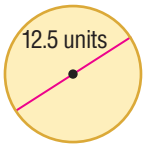
About 20 square miles of land is covered by the volcano.

Skill and Concept Check

1. Writing Math Explain how to estimate the area of any circle.

2. **OPEN ENDED** Find a circular object in your classroom or home. Estimate and then find the actual area of the object.

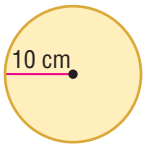
3. **FIND THE ERROR** Whitney and Crystal are finding the circle's area. Who is correct? Explain.

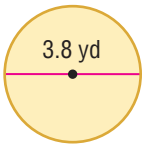


Whitney	Crystal
$A \approx 3.14 \times (12.5)^2$	$A \approx 3.14 \times (6.25)^2$

GUIDED PRACTICE

Find the area of each circle to the nearest tenth. Use 3.14 for π.

4. 

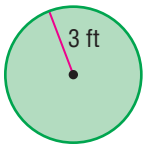
5. 

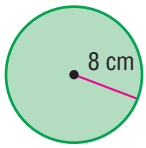
6. 

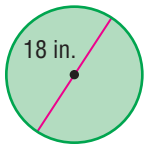
7. **SCIENCE** An earthquake's epicenter is the point from which the shock waves radiate. What is the area of the region affected by an earthquake whose shock waves radiated 29 miles from its epicenter?

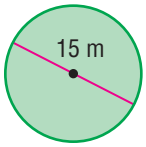
Practice and Applications

Find the area of each circle to the nearest tenth. Use 3.14 for π.

8. 

9. 

10. 

11. 

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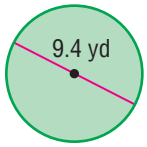
13. 

HOMEWORK HELP

For Exercises	See Examples
8–15	1, 2
16–18	3

Extra Practice
See pages 622, 637.

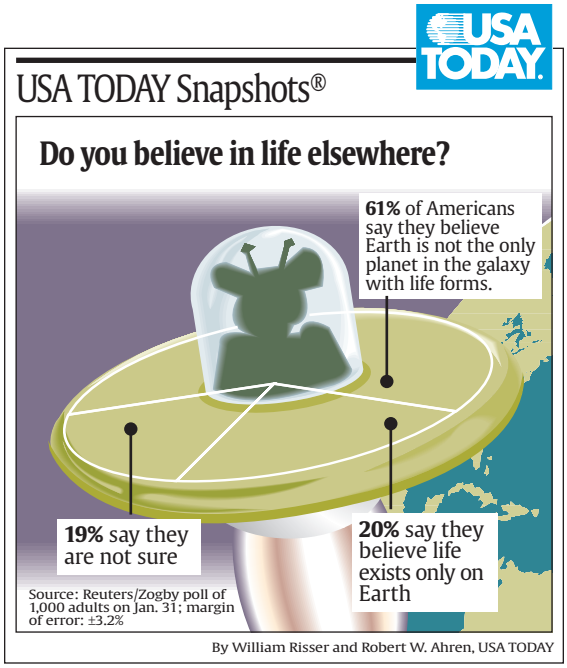
14. What is the area of a circle whose radius is 7.75 meters?

15. Find the area of a circle with a diameter of $175\frac{3}{8}$ feet.

16. **WRESTLING** A wrestling mat is a square mat measuring 12 meters by 12 meters. Within the square, there is a circular ring whose radius is 4.5 meters. Find the area within the circle to the nearest tenth.

Data Update Find the area of each circle that appears on a regulation hockey rink. How do the areas of these circles compare to the one on a wrestling mat? Visit msmath1.net/data_update to learn more.

17. **SCHOOL** Suppose you are preparing a report on people's beliefs in space aliens. You redraw the circle graph shown at the right on the report cover. When redrawn, the graph has a diameter of 9.5 inches. Find the area of the section of the graph that represents the 20% section to the nearest tenth.

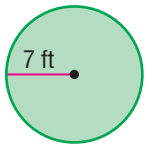


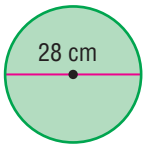
18. **TOOLS** A sprinkler that sprays water in a circular area can be adjusted to spray up to 30 feet. What is the maximum area of lawn that can be watered by the sprinkler?

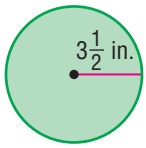
19. **CRITICAL THINKING** Suppose you double the radius of a circle. How is the area affected?

EXTENDING THE LESSON The fraction $\frac{22}{7}$ can also be used for π.

Find the area of each circle. Use $\frac{22}{7}$ for π.

20. 

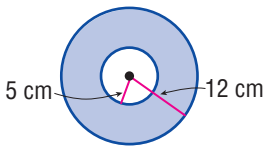
21. 

22. 

Spiral Review with Standardized Test Practice

23. **SHORT RESPONSE** Find the area of a circular hot tub cover whose diameter measures 6.5 feet. Round to the nearest tenth.

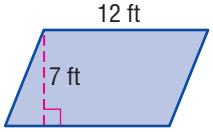
24. **MULTIPLE CHOICE** Find the area of the shaded region of the figure shown. Use 3.14 for π.



(A) 53.38 cm^2 (B) 373.66 cm^2
(C) 452.16 cm^2 (D) 530.66 cm^2

25. **GEOMETRY** What is the area of a triangle with a base 8 meters long and a height of 14 meters? (Lesson 14-2)

26. **GEOMETRY** Find the area of the parallelogram at the right. Round to the nearest tenth if necessary. (Lesson 14-1)



GETTING READY FOR THE NEXT LESSON

BASIC SKILL **Sketch each object listed.**

27. ice cream cone 28. shoe box 29. drinking straw

14-3b HANDS-ON LAB

A Follow-Up of Lesson 14-3

Making Circle Graphs

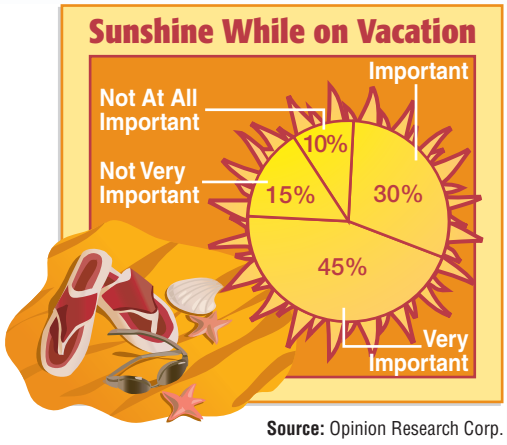
What You'll LEARN

Make circle graphs.

Materials

- colored pencils
- ruler
- compass
- protractor
- calculator

"How important is sunny weather in a vacation location?" The circle graph at the right shows how people responded to this question.



Source: Opinion Research Corp.

1. What percent of the people said that having sunshine while on vacation was not at all important?
2. What percent is represented by the whole circle graph? How many degrees are in the circle?
3. Explain when a circle graph is the best choice to display a set of data.

In this lab, you will learn to make circle graphs.

ACTIVITY *Work with a partner.*

A group of teenagers were asked to name their top priority for the school year. The results are shown at the right. Display the data in a circle graph.

Top Priorities for School Year

Top Priority	Percent
Sports	12.5%
Good Grades	50%
Friends	25%
Boyfriend/Girlfriend	12.5%

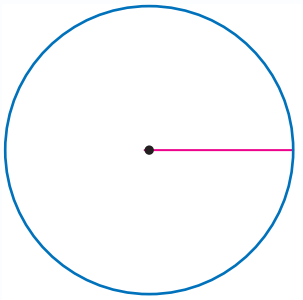
STEP 1 Find the number of degrees for each percent. To do this, first write each percent as a decimal. Then multiply each decimal by 360, the total number of degrees in a circle graph.

Percent to Decimal	Multiply by 360
$12.5\% \rightarrow 0.125$	$0.125 \times 360 = 45$
$50\% \rightarrow 0.50$	$0.50 \times 360 = 180$
$25\% \rightarrow 0.25$	$0.25 \times 360 = 90$
$12.5\% \rightarrow 0.125$	$0.125 \times 360 = 45$

The sum should always be 360.

The results are the number of degrees in the corresponding sections of the circle graph.

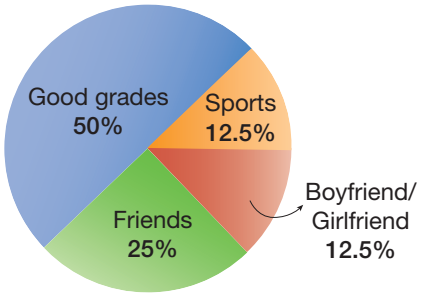
STEP 2 Use a compass to draw a circle with at least a 1-inch radius. Draw the radius with the ruler.



STEP 3 Use a protractor to draw an angle for the *Sports* section of the graph. Repeat Steps 1–3 for each category.

STEP 4 Shade each section of the graph. Then give the graph a title.

Top Priorities for School Year



$50\% = \frac{1}{2}$ so 50% is $\frac{1}{2}$ of the graph.

$25\% = \frac{1}{4}$ so 25% is $\frac{1}{4}$ of the graph.

$12.5\% = \frac{1}{8}$ so 12.5% is $\frac{1}{8}$ of the graph.

Your Turn **Display each set of data in a circle graph.**

a.

Time Spent Playing Video Games	
Time (h)	**Percent**
0–1	35%
1–2	10%
2–3	25%
3 or more	30%

b.

Time Film Stays in Camera Before Being Developed	
Time (months)	**Percent**
0–6	45%
6–12	37.5%
13–18	12.5%
don't know	5%

Writing Math

1. **Compare** each circle graph to its corresponding table. Does the graph or table display the data more clearly? Explain.
2. **Examine** each data set you displayed. Explain how each set of data compares part to whole relationships.
3. **Give an example** of a data set that *cannot* be represented by a circle graph. What type of graph would you use to best represent the data set?
4. **Explain** how the area of a circle is related to making a circle graph.

CHAPTER 14

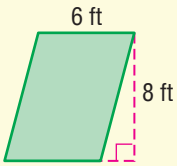
Mid-Chapter Practice Test

Vocabulary and Concepts

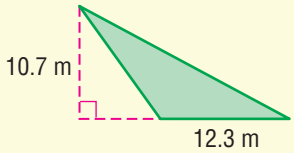
1. **Write** in words the formula for the area of a parallelogram. (Lesson 14-1)
2. **Explain** the relationship between the radius and the diameter of a circle. (Lesson 14-3)

Skills and Applications

Find the area of each figure. Round to the nearest tenth if necessary. (Lessons 14-1 and 14-2)

3. 

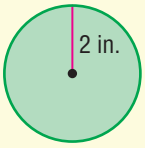
4. 

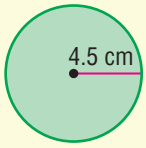
5. 

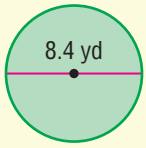
6. What is the measure of the area of a parallelogram whose base is $5\frac{2}{5}$ feet and whose height is $7\frac{1}{2}$ feet? (Lesson 14-1)

7. **BOATS** A sailboat has a triangular sail whose base is 10.5 feet and whose height is 30.75 feet. What is the area of the sail? (Lesson 14-2)

Find the area of each circle to the nearest tenth. Use 3.14 for π. (Lesson 14-3)

8. 

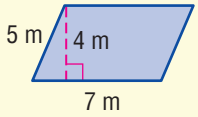
9. 

10. 

11. Find the area of a circle whose diameter is 10.5 inches. Round to the nearest tenth. (Lesson 14-3)

Standardized Test Practice

12. **MULTIPLE CHOICE** Which expression gives the area of the figure? (Lesson 14-1)



(A) 5×4 (B) 5×7
(C) 4×7 (D) $5 \times 4 \times 7$

13. **SHORT RESPONSE** A therapy pool is circular in shape. If the diameter is 9 meters, how much material is needed to make a cover for the pool? (Lesson 14-3)

The Game Zone

A Place To Practice Your Math Skills

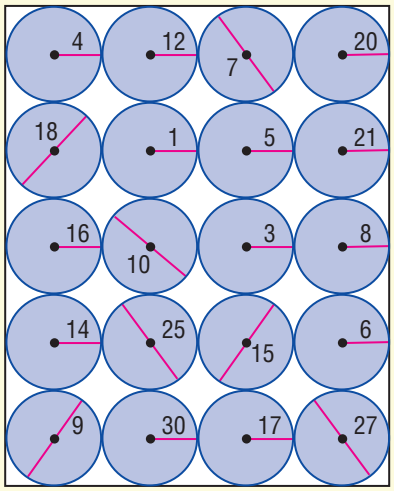
Time's Up for Circles

GET READY!

Players: five
Materials: poster board, compass, number cube, 1-minute timer

GET SET!

- Use a compass to draw the game board shown at the right.
- Choose one player to be the official timekeeper and answer checker.
- Divide into teams of two players.



GO!

- One player rolls a number cube onto the poster board.
- The player's team member has one minute to find the area of the circle on which the number cube lands.
- The answer checker checks the response and awards 5 points for a correct answer.
- The other team takes its turn.
- **Who Wins?** The team with the highest total score after five rounds wins.

14-4 Three-Dimensional Figures

What You'll LEARN

Identify three-dimensional figures.

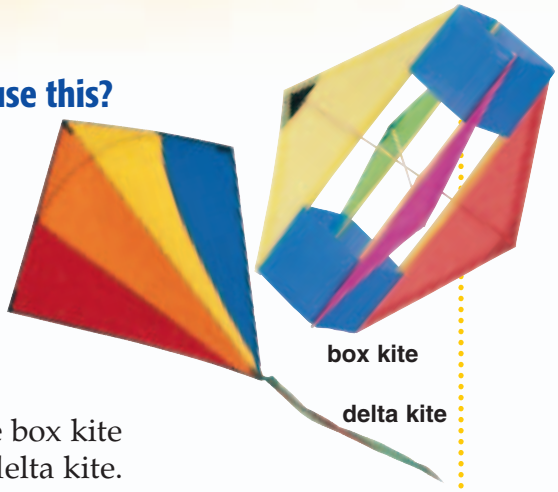
NEW Vocabulary

three-dimensional figure
face
edge
lateral face
vertex (vertices)
prism
base
pyramid
cone
cylinder
sphere
center

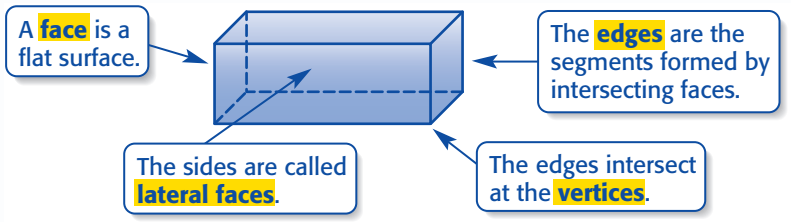
WHEN am I ever going to use this?

KITES A box kite and a delta kite are shown.

1. What shape does the delta kite resemble?
2. Name the shape that each side of the box kite resembles.
3. Describe how the shape of the box kite differs from the shape of the delta kite.



Many common shapes are **three-dimensional figures**. That is, they have length, width, and depth (or height). Some terms associated with three-dimensional figures are face, edge, vertex, and lateral face.



Two types of three-dimensional figures are prisms and pyramids.

Noteables™

Key Concept: Prisms and Pyramids

Prism

- Has at least three lateral faces that are rectangles.
- The top and bottom faces are the **bases** and are parallel.
- The shape of the base tells the name of the prism.

Rectangular prism

Triangular prism

Square prism or cube

Pyramid

- Has at least three lateral faces that are triangles and only one base.
- The base can be shaped like any closed figure with three or more sides.
- The shape of the base tells the name of the pyramid.

Triangular pyramid

Square pyramid

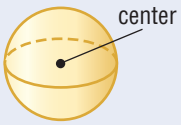
STUDY TIP

Three-Dimensional Figures In three-dimensional figures, dashed lines are used to indicate edges that are hidden from view.

CONTENTS

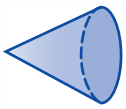
Some three-dimensional figures have curved surfaces.

Noteables™ **Key Concept: Cones, Cylinders, and Spheres**

Cone	• Has only one base. • The base is a circle. • Has one vertex and no edges.	
Cylinder	• Has only two bases. • The bases are circles. • Has no vertices and no edges.	
Sphere	• All of the points on a sphere are the same distance from the **center**. • No faces, bases, edges, or vertices.	

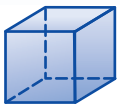
EXAMPLES Identify Three-Dimensional Figures

Identify each figure.

1. 

one circular base, no edge, and no vertex

The figure is a cone.

2. 

All of the faces are squares.

The figure is a cube.

Your Turn

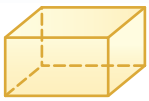
a. Identify the figure shown at the right. 

Skill and Concept Check

1. **Determine** the number of vertices for each figure.

a. 

b. 

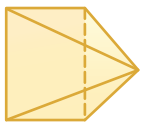
c. 

d. 

2. Writing Math Explain the difference between a two-dimensional and a three-dimensional figure.

GUIDED PRACTICE

Identify each figure.

3. 

4. 

5. **FOOD** Draw the figure that represents a can of soup. Then identify the figure.

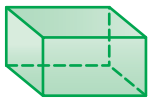
Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
6–9	1, 2

Extra Practice
See pages 623, 637.

Identify each figure.

6. 

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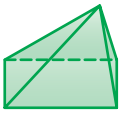
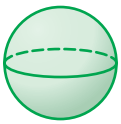
8. 

9. 

10. **SCHOOL** Draw the figure that represents a textbook. What is the name of this figure?

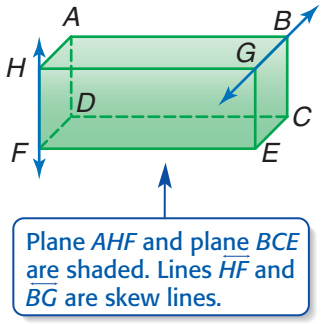
11. **SPORTS** Megaphones are used to intensify or direct the voice. Sketch the figure shown by a megaphone. Explain why it is not identified as a cone.

CRITICAL THINKING **For Exercises 12 and 13, draw figures to support your answer.**

12. What type of pyramid has exactly four faces?

13. What figure is formed if only the height of a cube is increased?

EXTENDING THE LESSON A *plane* is a flat surface that extends in all directions. The faces of a prism are parts of a plane. Two lines that are not in the same plane and do not intersect are *skew lines*.



14. Identify two other planes in the rectangular prism. Three vertices are needed to name a plane.

15. Name two other pairs of lines that are skew lines.

Spiral Review with Standardized Test Practice

16. **MULTIPLE CHOICE** The base of a cone is a ___?___.

Ⓐ triangle Ⓑ circle Ⓒ radius Ⓓ rectangle

17. **MULTIPLE CHOICE** Identify the figure shown.

Ⓕ triangular pyramid Ⓖ square pyramid
Ⓗ rectangular pyramid Ⓘ triangular prism

Find the area of each circle described to the nearest tenth. Use 3.14 for π. (Lesson 14-3)

18. radius: 22 in.

19. diameter: 6 m

20. diameter: 4.6 ft

21. **GEOMETRY** What is the area of a triangle whose base is 52 feet and whose height is 38 feet? (Lesson 14-2)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Multiply. Round to the nearest tenth.** (Lesson 4-2)

22. $8.2 \times 4.8 \times 2.1$

23. $5.9 \times 1.0 \times 7.3$

24. $1.0 \times 0.9 \times 1.3$

14-4b HANDS-ON LAB

A Follow-Up of Lesson 14-4

Three-Dimensional Figures

What You'll LEARN

Draw three-dimensional figures.

It is often helpful to draw a three-dimensional figure when trying to solve a problem.

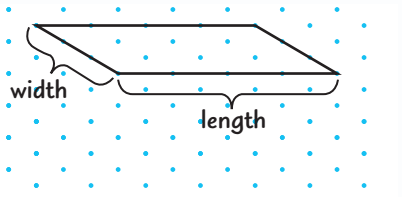
Materials

- isometric dot paper
- ruler

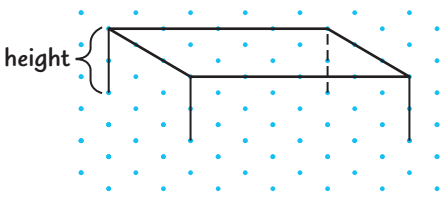
ACTIVITY *Work with a partner.*

Use isometric dot paper to sketch a rectangular prism with length 4 units, height 2 units, and width 3 units.

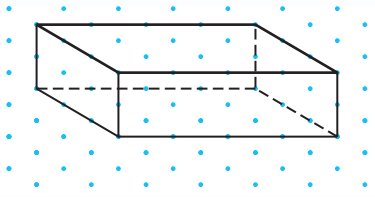
STEP 1 Draw a parallelogram with sides 4 units and 3 units. This is the top of the prism.



STEP 2 Start at one vertex. Draw a line passing through two dots. Repeat for the other three vertices. Draw the hidden edges as dashed lines.



STEP 3 Connect the ends of the lines to complete the prism.



Writing Math

1. **Explain** which faces are the bases of the prism.
2. Use isometric dot paper to draw each figure.
   a. a cube with length, width, and height of 3 units
   b. a rectangular prism with length 4 units, width 2 units, and height 2 units
3. How would you draw a prism with a triangular base?
4. **Explain** why you think isometric dot paper is used to draw a three-dimensional object.
5. Suppose you need to draw a three-dimensional representation of a sphere. Do you think this method would work? Explain.

14-5a Problem-Solving Strategy

A Preview of Lesson 14-5

Make a Model

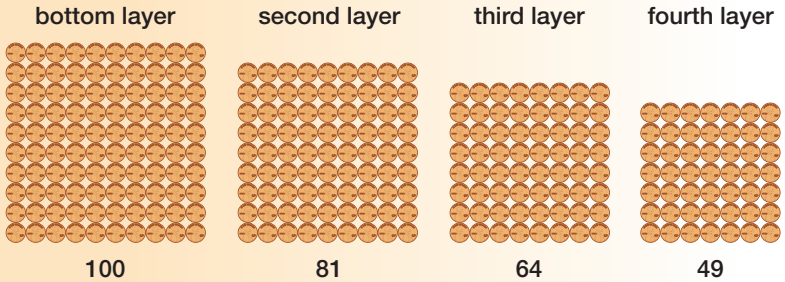
What You'll LEARN

Solve problems by making a model.

Hey Jaime, one of our first jobs at the grocery store is to stack oranges in the shape of a square pyramid. The base of the pyramid should have 100 oranges and one orange needs to be on top.

We have 400 oranges, Patrick. Is that enough? Let's **make a model** to find out!

Explore	The oranges are to be stacked in the form of a square pyramid. There are to be 100 oranges on the base and one orange at the top. You need to know how many oranges are needed to make the pyramid.
Plan	Make a model to find the number of oranges needed to make the pyramid.
Solve	Use pennies to model the oranges. Begin with 100 pennies for the bottom layer. For each consecutive layer, one penny is placed where four pennies meet. Continue this pattern until one penny is on the top layer.



By continuing the pattern, you will find that 100 + 81 + 64 + 49 + 36 + 25 + 16 + 9 + 4 + 1 or 385 oranges will be needed. So, we have enough.

Examine	Stack the pennies into a square pyramid with 100 pennies on the bottom and continue until one penny is on top. The result is 385.

Analyze the Strategy

1. **Tell** how making a model helped the students solve the problem.
2. **Write** a problem that can be solved by making a model.

John Evans

CONTENTS

Apply the Strategy

Solve. Use the make a model strategy.

3. **CRAFTS** Cory is designing a stained glass window made of triangle pieces of glass. If the window frame is 3 feet by 4 feet and the height and base of the triangular pieces are 4 inches long, how many triangles are needed to fill the window?

4. **SALES** Karen is making a pyramid-shaped display of cereal boxes. The bottom layer of the pyramid has six boxes. If there is one less box in each layer and there are five layers in the pyramid, how many boxes will Karen need to make the display?

Mixed Problem Solving

Solve. Use any strategy.

5. **BOOKS** A bookstore arranges its best-seller books in the front window. In how many different ways can four best-seller books be arranged in a row?

6. **MONEY** Mrs. Rivas works in sales. Her base salary is $650 per week, and she makes a 5% commission on her sales. What is Mrs. Rivas' salary for four weeks if she has $8,000 in sales?

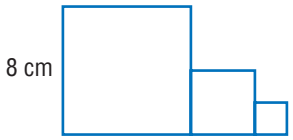
7. **PATTERNS** Draw the next figure.

8. **ART** The sixth grade class is planning a field trip to an art museum. There are 575 students in the sixth grade. If each bus holds 48 people, about how many buses will they need?

9. **MONEY** How many hats can be purchased with $90 if the hats can only be bought in pairs?

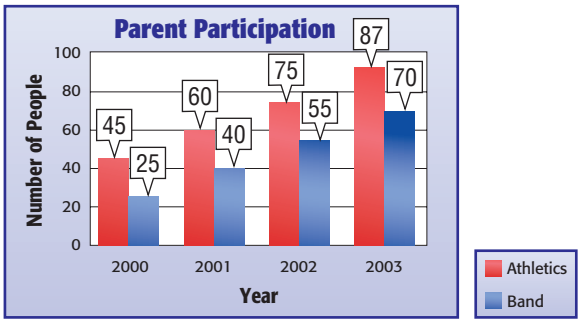
10. **FOOD** Robert bought 3 gallons of ice cream for a birthday party. If each serving size is about $\frac{1}{3}$ cup, how many servings will there be?

11. **GEOMETRY** The sides of each square in the figure are twice as long as the square on its immediate right. What is the perimeter of the entire figure?



12. **GEOMETRY** A rectangular prism is made using exactly 8 cubes. Find the length, width, and height of the prism.

13. **STANDARDIZED TEST PRACTICE**
The graph below shows the number of parents that have participated in the booster organizations at Rancher Heights Middle School. If the trend continues, about how many parents can be expected to participate in the band booster organization in 2005?



(A) 50 (B) 60 (C) 80 (D) 100

14-5 Volume of Rectangular Prisms

What You'll LEARN

Find the volume of rectangular prisms.

NEW Vocabulary

volume
cubic units

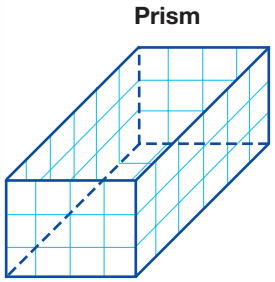
HANDS-ON Mini Lab

Materials

- centimeter grid paper
- tape
- centimeter cubes

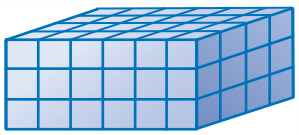
Work with a partner.

A rectangular prism and three different-sized groups of centimeter cubes are shown.



1. What are the dimensions of the prism?
2. Estimate how many of each group of cubes it will take to fill the prism. Assume that a group of cubes can be taken apart to fill the prism.
3. Use grid paper and tape to construct the prism. Then use centimeter cubes to find how many of each group of cubes it will take to fill the prism. Compare the results to your estimates.
4. Describe the relationship between the number of centimeter cubes that it takes to fill the prism and the product of the dimensions of the prism.

The amount of space inside a three-dimensional figure is the **volume** of the figure. Volume is measured in **cubic units**. This tells you the number of cubes of a given size it will take to fill the prism.



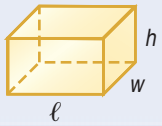
The volume of a rectangular prism is related to its dimensions.

Noteables™ **Key Concept: Volume of a Rectangular Prism**

Words The volume V of a rectangular prism is the product of its length ℓ, width w, and height h.

Symbols $V = \ell wh$

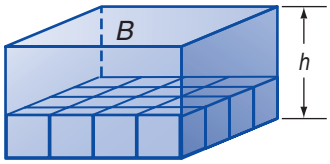
Model



READING Math

Volume Measurement
A volume measurement can be written using abbreviations and an exponent of 3.
For example:
cubic units = units^3
cubic inches = in^3
cubic feet = ft^3
cubic meters = m^3

Another method you can use to find the volume of a rectangular prism is to multiply the area of the base (B) by the height (h).



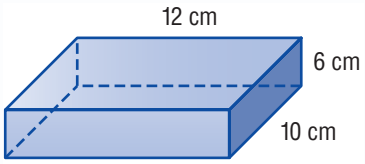
$V = Bh$

- B: area of the base, or the number of cubes needed to cover the base
- h: number of rows of cubes needed to fill the prism

EXAMPLE 1 Find the Volume of a Rectangular Prism

Find the volume of the rectangular prism.

In the figure, $\ell = 12$ cm, $w = 10$ cm, and $h = 6$ cm.



Method 1 Use $V = \ell wh$.

$V = \ell wh$

$V = 12 \times 10 \times 6$

$V = 720$

The volume is 720 cm^3.

Method 2 Use $V = Bh$.

B, or the area of the base, is 10×12 or 120 square centimeters.

$V = Bh$

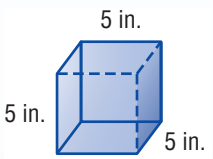
$V = 120 \times 6$

$V = 720$

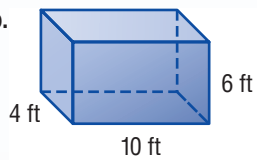
The volume is 720 cm^3.

Your Turn **Find the volume of each rectangular prism.**

a.



b.



REAL-LIFE MATH

FOOD The largest box of popcorn in the U.S. measured about 52.6 feet long and 10.1 feet wide. The average depth of the popcorn was 10.2 feet.

Source: *Guinness Book of Records*

EXAMPLE 2 Use Volume to Solve a Problem

FOOD **Use the information at the left. Find the approximate amount of popcorn that was contained within the popcorn box.**

To find the amount of popcorn, find the volume.

Estimate $50 \times 10 \times 10 = 5{,}000$

$V = \ell wh$ Volume of a rectangular prism

$V = 52.6 \times 10.1 \times 10.2$ $\ell = 52.6$, $w = 10.1$, $h = 10.2$

$V = 5{,}418.852$ Use a calculator.

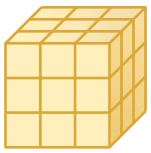
The box contained about 5,419 cubic feet of popcorn.

Compared to the estimate, the answer is reasonable.

Skill and Concept Check

1. Writing Math Explain why cubic units are used to measure volume instead of linear units or square units.

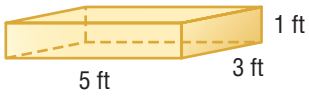
2. **GEOMETRY SENSE** Visualize the three-dimensional figure shown at the right. How many of the cubes would show only 2 outside faces?

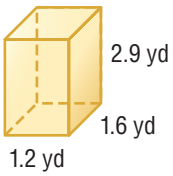


3. **OPEN ENDED** Draw a box with a volume of 24 cubic units.

GUIDED PRACTICE

Find the volume of each figure. Round to the nearest tenth if necessary.

4. 

5. 

6. **CAVES** A cave chamber is 2,300 feet long, 1,480 feet wide, and at least 230 feet high everywhere in the cave. What is the minimum volume of the cave?

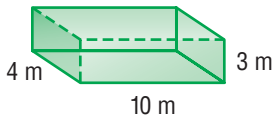
Practice and Applications

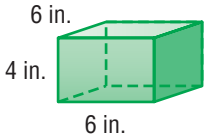
HOMEWORK HELP

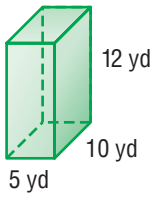
For Exercises	See Examples
7–12	1
13–14, 21	2

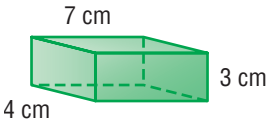
Extra Practice
See pages 623, 637.

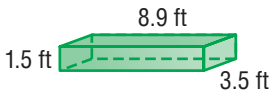
Find the volume of each figure. Round to the nearest tenth if necessary.

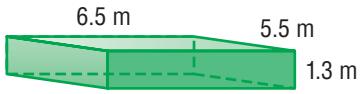
7. 

8. 

9. 

10. 

11. 

12. 

13. Find the volume to the nearest tenth of a rectangular prism having a length of 7.7 meters, width of 8.2 meters, and height of 9.7 meters.

14. What is the volume of a rectangular prism with a length of 10.3 feet, width of 9.9 feet, and height of 5.6 feet?

15. How many cubic feet are in 2 cubic yards?

16. How many cubic inches are in a cubic foot?

Replace each ● with $<$, $>$, or $=$ to make a true sentence.

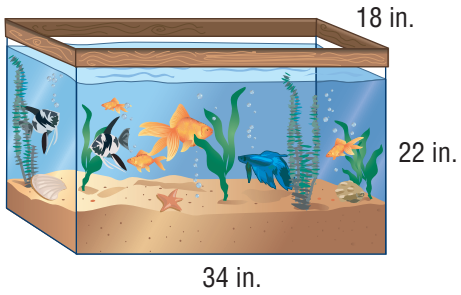
17. $1 \text{ ft}^3 \bullet 1 \text{ yd}^3$

18. $5 \text{ m}^3 \bullet 5 \text{ yd}^3$

19. $27 \text{ ft}^3 \bullet 1 \text{ yd}^3$

20. **WRITE A PROBLEM** Write a problem that can be solved by finding the volume of a rectangular prism.

21. **FISH** The fish tank shown is filled to a height of 15 inches. What is the volume of water in the tank?



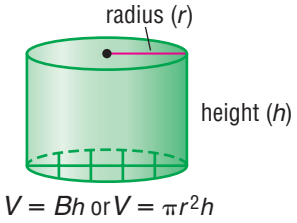
22. **RESEARCH** Use the Internet or another source to find the dimensions and volume of the largest fish tank at an aquarium or zoo in your state or in the United States.

23. **MULTI STEP** A storage container measures 3.5 inches in length, 5.5 inches in height, and 8 inches in width. What is the volume of the container if the width is decreased by 50%?

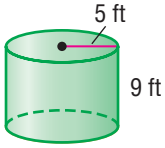
24. **CRITICAL THINKING** If all the dimensions of a rectangular prism are doubled, does the volume double? Explain.

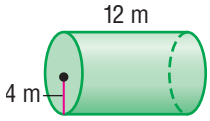
EXTENDING THE LESSON The volume of a cylinder is the number of cubic units needed to fill the cylinder.

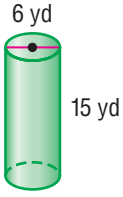
To find the volume V, multiply the area of the base B by the height h. Since the base is a circle, you can replace B in $V = Bh$ with πr^2 to get $V = \pi r^2 h$.



Find the volume of each cylinder to the nearest tenth. Use 3.14 for π.

25. 

26. 

27. 

Spiral Review with Standardized Test Practice

28. **MULTIPLE CHOICE** A rectangular prism has a volume of 288 cubic inches. Which dimensions could be the dimensions of the prism?

(A) 2 in., 4 in., 30 in.
(B) 2 in., 12 in., 12 in.
(C) 4 in., 72 in.
(D) 6 in., 8 in., 7 in.

29. **SHORT RESPONSE** Find the volume of the prism shown.

Draw each figure. (Lesson 14-4)

30. cylinder
31. triangular prism
32. sphere

33. **GEOMETRY** A circle has a radius that measures 5 yards. Estimate the area of the circle. (Lesson 14-3)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Add.** (Lesson 3-5)

34. 12.7 + 6.9 + 13.9
35. 19.0 + 1.5 + 17.8
36. 8.1 + 4.67 + 25.8

14-6a HANDS-ON LAB

A Preview of Lesson 14-6

Using a Net to Build a Cube

In this lab, you will make a two-dimensional figure called a **net** and use it to build a three-dimensional figure.

What You'll LEARN

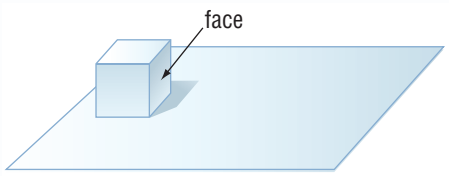
Build a three-dimensional figure from a net and vice versa.

Materials

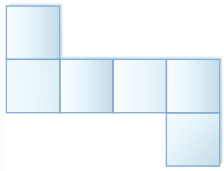
- cube
- scissors
- paper

ACTIVITY *Work with a partner.*

STEP 1 Place a cube on paper as shown. Trace the base of the cube, which is a square.

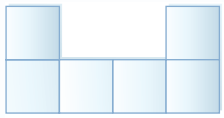


STEP 2 Roll the cube onto another side. Continue tracing each side to make the figure shown. This two-dimensional figure is called a net.



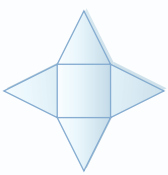
STEP 3 Cut out the net. Then build the cube.

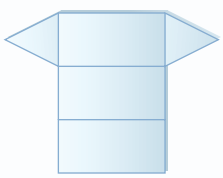
STEP 4 Make a net like the one shown. Cut out the net and try to build a cube.

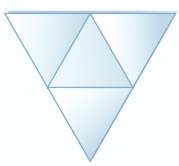


Writing Math

1. **Explain** whether both nets formed a cube. If not, describe why the net or nets did not cover the cube.
2. **Draw** three other nets that will form a cube and three other nets that will not form a cube. Describe a pattern in the nets that do form a cube.
3. **Draw** a net for a rectangular prism. Explain the difference between this net and the nets that formed a cube.
4. **Tell** what figure would be formed by each net. Explain.

a. 

b. 

c. 

14-6 Surface Area of Rectangular Prisms

What You'll LEARN

Find the surface areas of rectangular prisms.

NEW Vocabulary

surface area

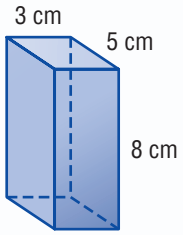
HANDS-ON Mini Lab

Materials

- ruler
- centimeter grid paper
- scissors
- calculator
- tape

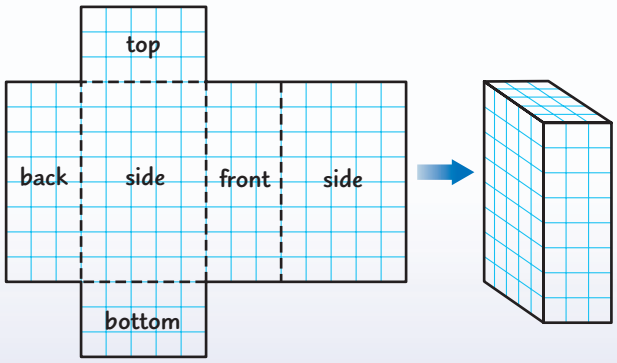
Work with a partner.

You can use a net to explore the sum of the areas of the faces of the prism shown.



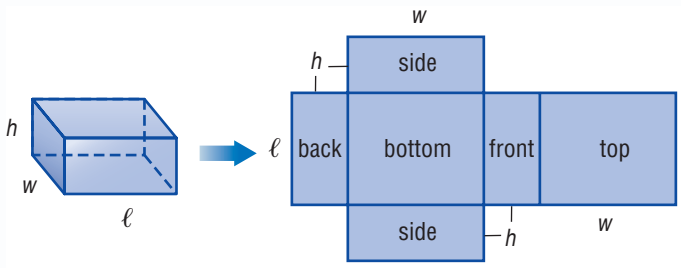
STEP 1 Draw and cut out a net of the prism.

STEP 2 Fold along the dashed lines. Tape the edges.



1. Find the area of each face of the prism.
2. What is the sum of the areas of the faces of the prism?
3. What do you notice about the area of opposite sides of the prism? How could this simplify finding the sum of the areas?

The sum of the areas of all the faces of a prism is called the **surface area** of the prism.



top and bottom: $(\ell \times w) + (\ell \times w) = 2\ell w$
front and back: $(\ell \times h) + (\ell \times h) = 2\ell h$
two sides: $(w \times h) + (w \times h) = 2wh$

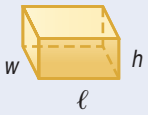
Sum of areas of faces $= 2\ell w + 2\ell h + 2wh$

Noteables™ Key Concept: Surface Area of Rectangular Prisms

Words The surface area S of a rectangular prism with length ℓ, width w, and height h is the sum of the areas of the faces.

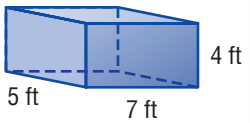
Symbols $S = 2\ell w + 2\ell h + 2wh$

Model



EXAMPLE 1 Find the Surface Area of a Rectangular Prism

Find the surface area of the rectangular prism.



Find the area of each face.

top and bottom

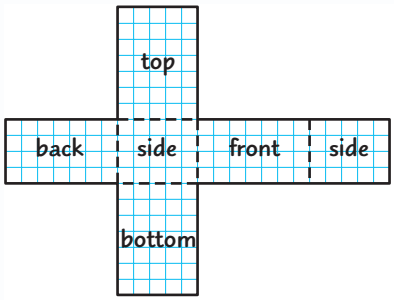
$2(\ell w) = 2(7 \times 5) = 70$

front and back

$2(\ell h) = 2(7 \times 4) = 56$

two sides

$2(wh) = 2(5 \times 4) = 40$

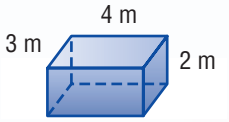


Add to find the surface area.

The surface area is 70 + 56 + 40 or 166 square feet.

Your Turn

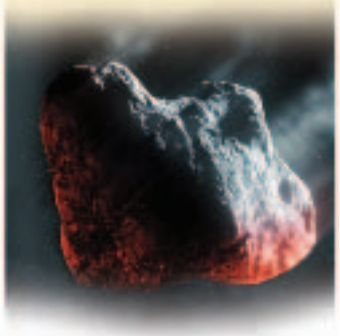
a. Find the surface area of the rectangular prism.



REAL-LIFE MATH

SPACE More than 10,000 asteroids have been cataloged and named. Around 200 asteroids have diameters of more than 100 kilometers.

Source: www.the-solar-system.net



Surface area can be applied to many real-life situations.

EXAMPLE 2 Use Surface Area to Solve a Problem

SPACE An asteroid measures about 21 miles long, 8 miles wide, and 8 miles deep. Its shape resembles a rectangular prism. What is the approximate surface area of the asteroid?

Use the formula for the surface area of a rectangular prism.

$S = 2\ell w + 2\ell h + 2wh$	Surface area of a prism
$S = 2(21 \times 8) + 2(21 \times 8) + 2(8 \times 8)$	$\ell = 21$, $w = 8$, $h = 8$
$S = 2(168) + 2(168) + 2(64)$	Simplify within parentheses.
$S = 336 + 336 + 128$	Multiply.
$S = 800$	Add.

The approximate surface area of the asteroid is 800 square miles.

Denis Scott/CORBIS

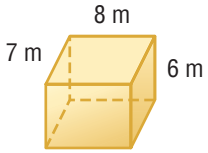
Skill and Concept Check

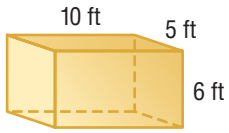
1. **Identify** each measure as *length*, *area*, *surface area*, or *volume*. Explain.
   a. the capacity of a lake
   b. the amount of land available to build a house
   c. the amount of wrapping paper needed to cover a box

2. **OPEN ENDED** Draw and label a rectangular prism that has a surface area greater than 200 square feet but less than 250 square feet.

GUIDED PRACTICE

Find the surface area of each rectangular prism.

3. 

4. 

5. Find the surface area of a rectangular prism that is 3.5 centimeters by 6.75 centimeters by 12 centimeters. Round to the nearest tenth.

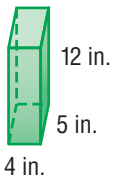
Practice and Applications

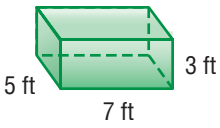
HOMEWORK HELP

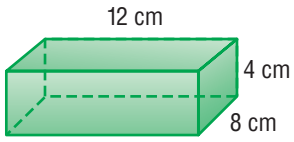
For Exercises	See Examples
6–11	1
12, 13–14	2

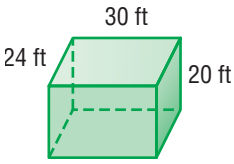
Extra Practice
See pages 623, 637.

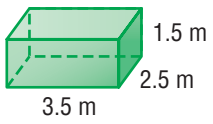
Find the surface area of each rectangular prism. Round to the nearest tenth if necessary.

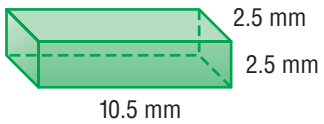
6. 

7. 

8. 

9. 

10. 

11. 

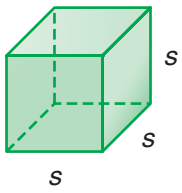
12. **AQUARIUMS** A shark petting tank is 20 feet long, 8 feet wide, and 3 feet deep. What is the surface area if the top of the tank is open?

FOOD **For Exercises 13–16, use the following information.**
Pretzels are to be packaged in the box shown.

13. Estimate the surface area of the box.
14. What is the actual surface area?
15. What is the surface area if the height is increased by 100%?
16. What is the surface area if the height is decreased by 50%?

17. **WRITE A PROBLEM** Write a problem involving a rectangular prism that has a surface area of 202 square inches.

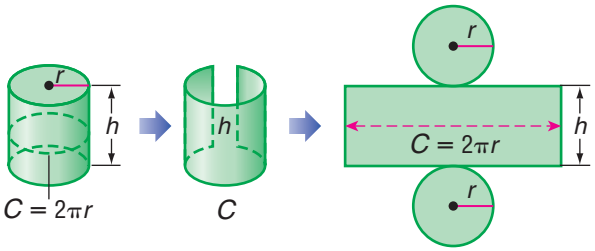
CRITICAL THINKING A cube is shown.



18. What is true about the area of the faces of a cube?

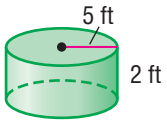
19. How could the formula $S = 2\ell w + 2\ell h + 2wh$ be simplified into a formula for the surface area of a cube?

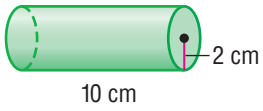
EXTENDING THE LESSON A net can also be used to show how to find the surface area of a cylinder.

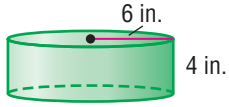


Faces	Area
top and bottom	$(\pi r^2) + (\pi r^2) = 2(\pi r^2)$
rectangular region	$(\ell \times w)$ or $2\pi rh$
Surface area = $2(\pi r^2) + 2\pi rh$	

Find the surface area of each cylinder. Round to the nearest tenth. Use 3.14 for π.

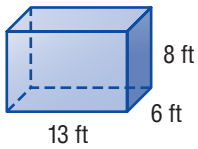
20. 

21. 

22. 

Spiral Review with Standardized Test Practice

23. **MULTIPLE CHOICE** Find the surface area of the prism shown.



Ⓐ 425 ft^2 Ⓑ 440 ft^2 Ⓒ 460 ft^2 Ⓓ 468 ft^2

24. **MULTIPLE CHOICE** Find the surface area of a cube whose sides measure 5.5 inches. Round to the neatest tenth.

Ⓕ 225.5 in^2 Ⓖ 181.5 in^2 Ⓗ 125.5 in^2 Ⓘ 30.3 in^2

25. **GEOMETRY** Find the volume of a rectangular prism whose sides measure 5 feet, 8 feet, and $10\frac{1}{2}$ feet. (Lesson 14-5)

26. **FOOD** Draw a figure that represents a cereal box. Then identify the figure. (Lesson 14-4)

INTERDISCIPLINARY PROJECT

Road Trip

Math and Geography It's time to complete your project. Use the data you have gathered about where you are going and what you will take to prepare a Web page or poster. Be sure to include all dimensions and volume calculations with your project.

WebQuest msmath1.net/webquest

CHAPTER 14

Study Guide and Review

Vocabulary and Concept Check

base (pp. 546, 564)
center (p. 565)
cone (p. 565)
cubic units (p. 570)
cylinder (p. 565)
edge (p. 564)
face (p. 564)
height (p. 546)
lateral face (p. 564)
prism (p. 564)
pyramid (p. 564)
sphere (p. 565)
surface area (p. 575)
three-dimensional figure (p. 564)
vertex (vertices) (p. 564)
volume (p. 570)

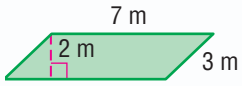
Choose the correct term to complete each sentence.

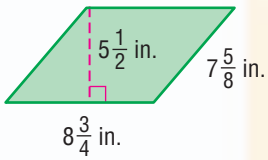
1. The flat surfaces of a three-dimensional figure are called (faces, vertices).
2. A (pyramid, cylinder) is a three-dimensional figure with one base where all other faces are triangles that meet at one point.
3. A three-dimensional figure with two circular bases is a (cone, cylinder).
4. The amount of space that a three-dimensional figure contains is called its (area, volume).
5. The total area of a three-dimensional object's faces and curved surfaces is called its (surface area, volume).

Lesson-by-Lesson Exercises and Examples

14-1 Area of Parallelograms (pp. 546–549)

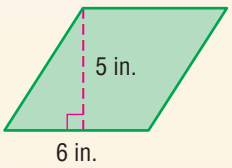
Find the area of each parallelogram. Round to the nearest tenth if necessary.

6. 

7. 

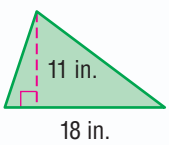
Example 1 **Find the area of the parallelogram.**

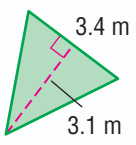
$A = bh$
$A = 6 \times 5$
$A = 30 \text{ in}^2$



14-2 Area of Triangles (pp. 551–554)

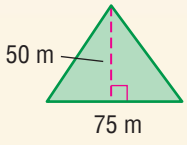
Find the area of each triangle. Round to the nearest tenth if necessary.

8. 

9. 

Example 2 **Find the area of the triangle.**

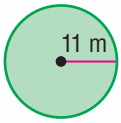
$A = \frac{1}{2}bh$
$A = \frac{1}{2}(75 \times 50)$
$A = 1{,}875 \text{ m}^2$



Mixed Problem Solving
For mixed problem-solving practice, see page 637.

14-3 Area of Circles (pp. 556–559)

Find the area of each circle to the nearest tenth. Use 3.14 for π.

10. 

11. 

12. **RIDES** The plans for a carousel call for a circular floor with a diameter of 40 feet. Find the area of the floor.

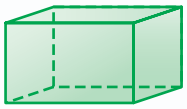
Example 3 **Find the area to the nearest tenth. Use 3.14 for π.**

$A = \pi r^2$ Area of a circle
$A \approx 3.14 \times 7^2$ Let $\pi = 3.14$ and $r = 7$.
$A \approx 3.14 \times 49$ Evaluate 7^2.
$A \approx 153.9$ Multiply.
The area is about 153.9 square centimeters.

14-4 Three-Dimensional Figures (pp. 564–566)

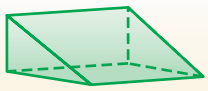
Identify each figure.

13. 

14. 

15. **SPORTS** What is the shape of a basketball?

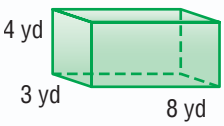
Example 4
Identify the figure.

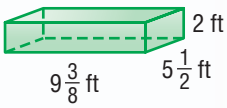


It has at least three rectangular lateral faces. The bases are triangles.
The figure is a triangular prism.

14-5 Volume of Rectangular Prisms (pp. 570–573)

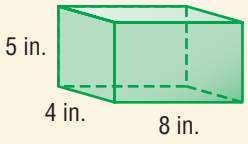
Find the volume of each figure. Round to the nearest tenth if necessary.

16. 

17. 

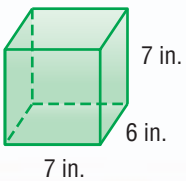
Example 5 **Find the volume of the prism.**

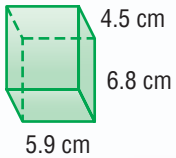
$V = \ell wh$
$V = 8 \times 4 \times 5$
$V = 160$
The volume is 160 cubic inches.



14-6 Surface Area of Rectangular Prisms (pp. 575–578)

Find the surface area of each rectangular prism. Round to the nearest tenth if necessary.

18. 

19. 

Example 6 **Find the surface area of the rectangular prism in Example 5.**

top and bottom: $2(8 \times 4)$ or 64
front and back: $2(8 \times 5)$ or 80
two sides: $2(4 \times 5)$ or 40
The surface area is $64 + 80 + 40$ or 184 square feet.

CHAPTER 14

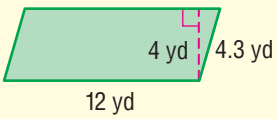
Practice Test

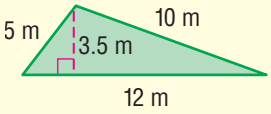
Vocabulary and Concepts

1. **Write** the formula for the area of a triangle.
2. **Define** *volume*.

Skills and Applications

Find the area of each figure. Round to the nearest tenth if necessary.

3. 

4. 

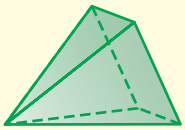
5. 

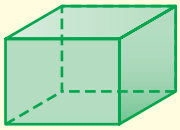
6. **TRAFFIC SIGN** A triangular yield sign has a base of 32 inches and a height of 30 inches. Find the area of the sign.

7. **GARDENING** A circular flowerbed has a radius of 2 meters. If you can plant 40 bulbs per square meter, how many bulbs should you buy?

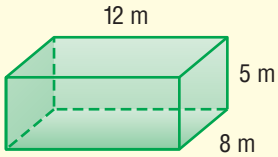
Identify each figure.

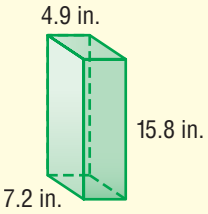
8. 

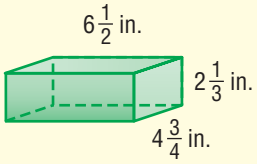
9. 

10. 

Find the volume of each figure. Round to the nearest tenth if necessary.

11. 

12. 

13. 

14. **POOLS** A rectangular diving pool is 20 feet by 15 feet by 8 feet. How much water is required to fill the pool?

15. Find the surface area of the prism in Exercise 11.

Standardized Test Practice

16. **MULTIPLE CHOICE** Which expression gives the surface area of a rectangular prism with length ℓ, width w, and height h?

(A) $2\ell^2 + 2h^2 + 2w^2$

(B) $2\ell w + 2\ell h + 2wh$

(C) $2(\ell \times w \times h)$

(D) $2\ell(w + h)$

CHAPTER 14

Standardized Test Practice

PART 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. An airplane is flying at a height of 23,145.769 feet. Which of the following numbers is in the hundreds place? (Prerequisite Skill, p. 586)

Ⓐ 1 Ⓑ 3 Ⓒ 4 Ⓓ 6

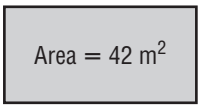
2. What is the best estimate for the total number of pounds of paper recycled by Ms. Maliqua's class? (Lesson 3-4)

Paper Recycling Drive	
Week	**Amount (lb)**
1	22.5
2	38.2
3	32.7
4	53.1

Ⓕ 130 lb Ⓖ 140 lb

Ⓗ 160 lb Ⓘ 170 lb

3. What could be the perimeter of the rectangle shown? (Lesson 4-5)



Ⓐ 13 m Ⓑ 20 m

Ⓒ 26 m Ⓓ 88 m

4. What is $3\frac{9}{16}$ expressed as an improper fraction? (Lesson 5-3)

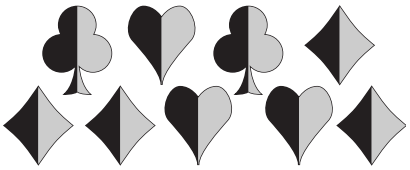
Ⓕ $\frac{48}{16}$ Ⓖ $\frac{43}{16}$

Ⓗ $3\frac{16}{9}$ Ⓘ $\frac{57}{16}$

5. What is the value of r in the equation $6r = 30$? (Lesson 9-4)

Ⓐ 0.2 Ⓑ 5 Ⓒ 24 Ⓓ 180

6. What is the ratio of the number of hearts to the total number of figures below? (Lesson 10-1)



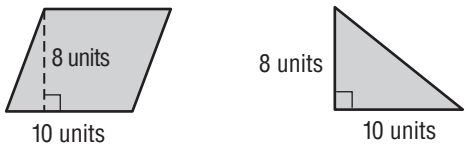
Ⓕ $\frac{2}{9}$ Ⓖ $\frac{1}{4}$ Ⓗ $\frac{1}{3}$ Ⓘ $\frac{4}{9}$

7. What is the area of a parallelogram with a base of 5 inches and a height of 3 inches? (Lesson 14-1)

Ⓐ 8 in^2 Ⓑ 15 in^2

Ⓒ 15 in. Ⓓ 16 in^2

8. What is a correct statement about the relationship between the figures shown? (Lesson 14-2)



Ⓕ The area of the parallelogram is the same as the area of the triangle squared.

Ⓖ The area of the parallelogram is three times the area of the triangle.

Ⓗ The area of the parallelogram is twice the area of the triangle.

Ⓘ The area of the parallelogram is $\frac{1}{2}$ the area of the triangle.

TEST-TAKING TIP

Question 7 When answer choices include units, be sure to select an answer choice that uses the correct units.

PART 2 Short Response/Grid In

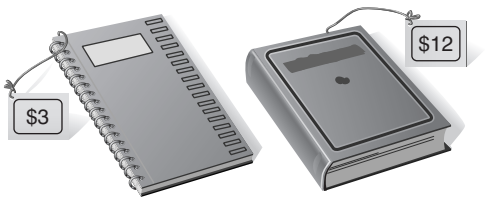
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. Kaley divides $3\frac{3}{5}$ pies among 9 people. How much of one pie will each person get? (Lesson 7-5)

10. Each serving of pizza is $\frac{1}{16}$ of a pizza. If $\frac{3}{4}$ of the pizza is left, how many servings are left? (Lesson 7-5)

11. What is the product of -7 and -12? (Lesson 8-4)

12. Elias bought the following items.



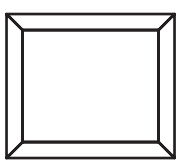
If the rate of sales tax that he paid was 6%, how much sales tax did he pay? (Lesson 10-7)

13. Find the probability that a randomly thrown dart will land in one of the squares labeled C. (Lesson 11-4)

A	B	B	B
B	A	C	B
C	A	B	C
B	C	B	A

14. How many inches are in 3 yards? (Lesson 12-1)

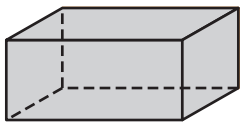
15. How many lines of symmetry does the figure shown have? (Lesson 13-5)



16. Find the area of a triangle that has a base of 12 inches and a height of 4 inches. (Lesson 14-2)

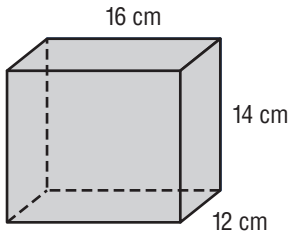
17. What is the approximate area of a circle with a radius of 10 meters? (Lesson 14-3)

18. How many faces does the rectangular prism have? (Lesson 14-4)



19. Write the formula that could be used to find the volume of a rectangular prism. (Use ℓ for length, w for width, h for height, and V for volume.) (Lesson 14-5)

20. What is the surface area of the rectangular prism? (Lesson 14-6)



PART 3 Extended Response

Record your answers on a sheet of paper. Show your work.

21. Shane built a figure using centimeter cubes. The figure stood 4 cubes high and covered a 12-centimeter by 8-centimeter area of the floor.

a. What area of the floor did the figure cover? (Lesson 14-1)

b. What is the volume of the figure? (Lesson 14-5)

c. Draw Shane's structure. (Lesson 14-5)