

# Probability



## “What does soccer have to do with math?”

Suppose a soccer player scored 6 goals in her last 15 attempts. You can use this ratio to find the probability of the soccer player scoring on her next attempt. In addition, you can use the probability to predict the number of goals she will score in her next 50 attempts.

You will solve problems about sports such as soccer, football, and golf in Lessons 11-3, 11-4, and 11-5.

# GETTING STARTED

## ► Diagnose Readiness

Take this quiz to see if you are ready to begin Chapter 11. Refer to the lesson number in parentheses for review.

### Vocabulary Review

Complete each sentence.

1. A fraction is in    ? form when the GCF of the numerator and denominator is one. (Lesson 5-2)
2. A    ? is a comparison of two numbers by division. (Lesson 10-1)

### Prerequisite Skills

Write each fraction in simplest form.

(Lesson 5-2)

- |                    |                   |
|--------------------|-------------------|
| 3. $\frac{15}{40}$ | 4. $\frac{7}{63}$ |
| 5. $\frac{21}{30}$ | 6. $\frac{8}{28}$ |
| 7. $\frac{9}{21}$  | 8. $\frac{9}{45}$ |

Multiply. Write in simplest form.

(Lesson 7-2)

- |  |                                       |
|--|---------------------------------------|
| 9. $\frac{5}{6} \times \frac{3}{10}$   | 10. $\frac{1}{8} \times \frac{7}{9}$  |
| 11. $\frac{11}{12} \times \frac{4}{5}$ | 12. $\frac{2}{3} \times \frac{9}{14}$ |
| 13. $\frac{2}{5} \times \frac{3}{4}$   | 14. $\frac{4}{7} \times \frac{2}{3}$  |

Solve each proportion. (Lesson 10-2)

- |                                    |                                   |
|------------------------------------|-----------------------------------|
| 15. $\frac{a}{6} = \frac{5}{15}$   | 16. $\frac{b}{9} = \frac{56}{84}$ |
| 17. $\frac{8}{y} = \frac{28}{42}$  | 18. $\frac{4}{c} = \frac{12}{30}$ |
| 19. $\frac{33}{44} = \frac{x}{12}$ | 20. $\frac{3}{8} = \frac{d}{24}$  |

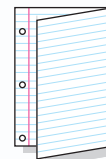
## FOLDABLES Study Organizer

**Probability** Make this Foldable to help you organize information about probability. Begin with one sheet of notebook paper.

### STEP 1

#### Fold

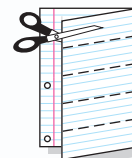
Fold the paper lengthwise to the holes.



### STEP 2

#### Unfold and Cut

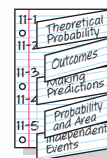
Unfold the paper and cut five equal tabs as shown.



### STEP 3

#### Label

Label lesson numbers and titles as shown.



## Noteables™

### Chapter Notes

Each time you find this logo throughout the chapter, use your *Noteables™*: *Interactive Study Notebook with Foldables™* or your own notebook to take notes. Begin your chapter notes with this Foldable activity.



**Readiness** To prepare yourself for this chapter with another quiz, visit [msmath1.net/chapter\\_readiness](http://msmath1.net/chapter_readiness)

**What You'll LEARN**

Explore experimental probability by conducting a simulation.

**Materials**

- 3 two-colored counters
- cups
- spinner

**Simulations**

A **simulation** is a way of acting out a problem situation. Simulations can be used to find probability, which is the chance something will happen. When you find a probability by doing an experiment, you are finding **experimental probability**.

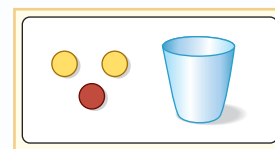
To explore experimental probability using a simulation, you can use these steps.

- Choose the most appropriate manipulative to aid in simulating the problem. Choose among counters, number cubes, coins, or spinners.
- Act out the problem for many trials and record the results to find an experimental probability.

**ACTIVITY** *Work with a partner.*

- 1** Use cups and counters to explore the experimental probability that at least two of three children in a family are girls.

**STEP 1** Place three counters in a cup and toss them onto your desk.



**STEP 2** Count the number of red counters. This represents the number of boys. The number of yellow counters represents the number of girls.

**STEP 3** Record the results in a table like the one shown.

Trial	Outcome		
1	B	B	G
2	B	G	G
3			
⋮			
50			

**STEP 4** Repeat Steps 1–3 for 50 trials.

Suppose 23 of the 50 trials have at least two girls. The experimental probability that at least two of the three children in a family are girls is  $\frac{23}{50}$  or 0.46.

**Your Turn**

- a. Describe a simulation to explore the experimental probability that two of five children in a family are boys. Then conduct your experiment. What is the experimental probability?

Spinners can also be used in simulations.

### ACTIVITY

*Work with a partner.*

- 1 The probability of the Hornets beating the Jets is 0.5. The probability of the Hornets beating the Flashes is 0.25. Find the experimental probability that the Hornets beat both the Jets and the Flashes.

**STEP 1** A probability of 0.5 is equal to  $\frac{1}{2}$ . This means that the Hornets should win 1 out of 2 games. Make a spinner as shown. Label one section "win" and the other section "lose".



**STEP 2** A probability of 0.25 is equal to  $\frac{1}{4}$ . This means that the Hornets should win 1 out of 4 games. Make a spinner as shown. Label one section "win" and the other sections "lose".



**STEP 3** Spin each spinner and record the results in a table like the one shown at the right. Repeat for 100 trials.

Trial	Outcome	
	Hornets and Jets	Hornets and Flashes
1	L	W
2	W	W
3		
⋮		
100		

**STEP 4** Use the results of the trials to write the ratio  $\frac{\text{beat both teams}}{100}$ . The ratio represents the experimental probability that the Hornets beat both teams.

### Your Turn

- b. The probability of rain on Monday is 0.75, and the probability of rain on Tuesday is 0.4. Describe a simulation you could use to explore the probability of rain on both days. Conduct your simulation to find the experimental probability of rain on both days.

## Writing Math

1. Explain experimental probability.
2. How is a simulation used to find the experimental probability of an event?

# Theoretical Probability

## What You'll LEARN

Find and interpret the theoretical probability of an event.

## NEW Vocabulary

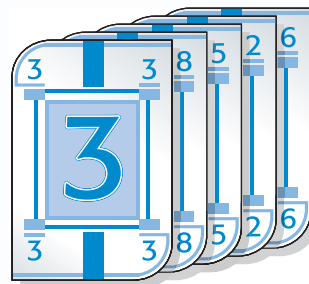
outcomes  
event  
theoretical probability  
complementary events

## REVIEW Vocabulary

**ratio:** comparison of two numbers by division  
(Lesson 10-1)

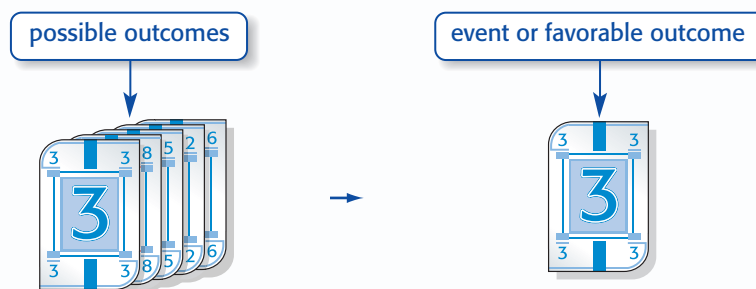
**WHEN** am I ever going to use this?

**GAMES** Drew and Morgan are playing cards. Morgan needs to draw a 3 in order to make a match and win the game. The cards shown are shuffled and placed facedown on the table.



1. Write a ratio that compares the number of cards numbered 3 to the total number of cards.
2. What percent of the cards are numbered 3?
3. Does Morgan have a good chance of winning? Explain.
4. What would happen to her chances of winning if cards 1, 4, 7, 9, and 10 were added to the cards shown?
5. What happens to her chances if only cards 3 and 8 are facedown on the table?

It is equally likely to select any one of the five cards. The player hopes to select a card numbered 3. The five cards represent the possible **outcomes**. The specific outcome the player is looking for is an **event**, or favorable outcome.



**Theoretical probability** is the chance that some event will occur. It is based on known characteristics or facts. You can use a ratio to find probability.

## READING in the Content Area

For strategies in reading this lesson, visit [msmath1.net/reading](http://msmath1.net/reading).

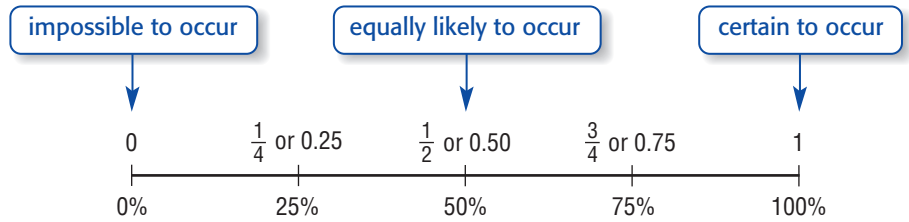
## Noteables

### Key Concept: Theoretical Probability

**Words** The theoretical probability of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.

**Symbols**  $P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

The probability that an event will occur is a number from 0 to 1, including 0 and 1. The closer a probability is to 1, the more likely it is to happen.

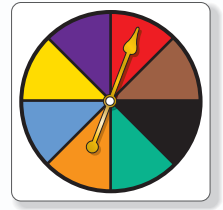


### READING Math

**Probability** The notation  $P(\text{red})$  is read the probability of landing on red.

### EXAMPLES Find Probability

There are eight equally likely outcomes on the spinner.



- 1 Find the probability of spinning red.

$$P(\text{red}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{1}{8}$$

The probability of landing on red is  $\frac{1}{8}$ , 0.125, or 12.5%.

- 1 Find the probability of spinning blue or yellow.

$$P(\text{blue or yellow}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{2}{8} \text{ or } \frac{1}{4} \text{ Simplify.}$$

The probability of landing on blue or yellow is  $\frac{1}{4}$ , 0.25, or 25%.

**Complementary events** are two events in which either one or the other must happen, but they cannot happen at the same time. An example is a coin landing on heads or not landing on heads. The sum of the probabilities of complementary events is 1.

### EXAMPLE Use Probability to Solve a Problem

- 1 **WEATHER** The morning newspaper reported a 20% chance of snow. What is the probability that it will not snow?

The two events are complementary. So, the sum of the probabilities is 1.

$$\begin{aligned} P(\text{snow}) + P(\text{not snowing}) &= 1 \\ 0.2 + P(\text{not snowing}) &= 1 && \text{Replace } P(\text{snow}) \text{ with } 0.2. \\ - 0.2 & && = - 0.2 \quad \text{Subtract } 0.2 \text{ from each side.} \\ \hline P(\text{not snowing}) &= 0.8 \end{aligned}$$



## Skill and Concept Check

- OPEN ENDED** Give an example of a situation in which the probability of an event occurring is 0.
- Writing Math** Explain what you can conclude about an event if its probability is 1.
- FIND THE ERROR** Laura and Lourdes are finding the probability of rolling a 5 on a number cube. Who is correct? Explain your reasoning.

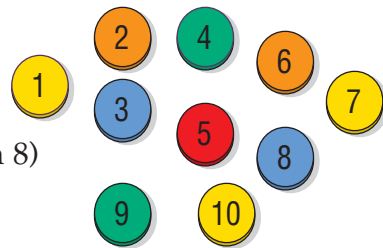
Laura  
Favorable: 5  
Possible: 1, 2, 3, 4, 5, 6  
 $P(5) = \frac{1}{6}$

Lourdes  
Favorable: 5  
Unfavorable: 1, 2, 3, 4, 6  
 $P(5) = \frac{1}{5}$

## GUIDED PRACTICE

A counter is randomly chosen. Find each probability. Write each answer as a fraction, a decimal, and a percent.

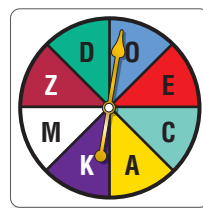
- $P(4)$
- $P(2 \text{ or } 5)$
- $P(\text{less than } 8)$
- $P(\text{greater than } 4)$
- $P(\text{prime})$
- $P(\text{not } 6)$
- SCHOOL** The probability of guessing the answer to a true-false question correctly is 50%. What is the probability of guessing the answer incorrectly?



## Practice and Applications

The spinner shown is spun once. Find each probability. Write each answer as a fraction, a decimal, and a percent.

- $P(Z)$
- $P(U)$
- $P(A \text{ or } M)$
- $P(C, D \text{ or } A)$
- $P(\text{vowel})$
- $P(\text{consonant})$



### HOMEWORK HELP

For Exercises	See Examples
11–25	1, 2
26	3

Extra Practice  
See pages 616, 634.

A number cube is rolled. Find the probability of each event. Write each answer as a fraction, a decimal, and a percent.

- $P(3)$
- $P(4 \text{ or } 6)$
- $P(\text{greater than } 4)$
- $P(\text{less than } 1)$
- $P(\text{even})$
- $P(\text{odd})$
- $P(\text{multiple of } 2)$
- $P(\text{not } 3 \text{ and not } 4)$
- $P(\text{not less than } 2)$

**WEATHER** For Exercises 26 and 27, use the following information.

A morning radio announcer reports that the chance of rain today is 85%.

- What is the probability that it will not rain?
- Should you carry an umbrella? Explain.

One marble is selected without looking from the bag shown. Write a sentence explaining how likely it is for each event to happen.



28.  $P(\text{green})$

29.  $P(\text{yellow})$

30.  $P(\text{purple})$

31.  $P(\text{blue})$

32. **SCHOOL** There are 160 girls and 96 boys enrolled at Grant Middle School. The school newspaper is randomly selecting a student to be interviewed. Find the probability of selecting a girl. Write as a fraction, a decimal, and a percent.

**CRITICAL THINKING** For Exercises 33 and 34, use the following information.

A spinner for a board game has more than three equal sections, and the probability of the spinner stopping on blue is 0.5.

33. Draw two possible spinners for the game.

34. Explain why each spinner drawn makes sense.

### EXTENDING THE LESSON

Another way to describe the chance of an event occurring is with odds. The *odds* in favor of an event is the ratio that compares the number of ways the event can occur to the ways that the event *cannot* occur.



odds of rolling a 3 or a 4 on a number cube  $\rightarrow 2 : 4$  or  $1 : 2$

Find the odds of each outcome if a number cube is rolled.

35. a 2, 3, 5, or 6

36. a number less than 3

37. an odd number

## Spiral Review with Standardized Test Practice

38. **MULTIPLE CHOICE** A number cube is rolled. What is the probability of rolling a composite number?

(A)  $\frac{1}{6}$

(B)  $\frac{1}{3}$

(C)  $\frac{1}{2}$

(D)  $\frac{2}{3}$

39. **SHORT RESPONSE** The probability of spinning a 5 on the spinner is  $\frac{2}{3}$ . What number is missing from the spinner?



40. Estimate 31% of 15. (Lesson 10-8)

Find the percent of each number. (Lesson 10-7)

41. 32% of 148

42. 6% of 25

### GETTING READY FOR THE NEXT LESSON

**BASIC SKILL** List all outcomes for each situation.

43. tossing a coin

44. rolling a number cube

45. selecting a month of the year

46. choosing a color of the American flag





**What You'll LEARN**

Compare experimental probability with theoretical probability.

**Materials**

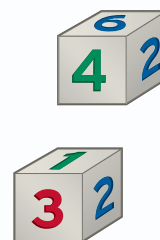
- 2 number cubes

**Experimental and Theoretical Probability****INVESTIGATE** *Work with a partner.*

In this lab, you will investigate the relationship between experimental probability and theoretical probability.

**STEP 1** The table shows all of the possible outcomes when you roll two number cubes. The highlighted outcomes are doubles.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)



Find the theoretical probability of rolling doubles.

**STEP 2** Copy the table shown. Then roll a pair of number cubes and record the results in the table. Write D for doubles and N for not doubles. Repeat for 30 trials.

Trials	Outcome
1	N
2	D
3	
⋮	
30	

**Writing Math**

1. Find the experimental probability of rolling doubles for the 30 trials. How does the experimental probability compare to the theoretical probability? Explain any differences.
2. Compare the results of your experiment with the results of the other groups in your class. Why do you think experimental probabilities usually vary when an experiment is repeated?
3. Find the experimental probability for the entire class's trials. How does the experimental probability compare to the theoretical probability?
4. **Explain** why the experimental probability obtained in Exercise 3 may be closer in value to the theoretical probability than the experimental probability in Exercise 1.

### What You'll LEARN

Find outcomes using lists, tree diagrams, and combinations.

### NEW Vocabulary

sample space  
tree diagram

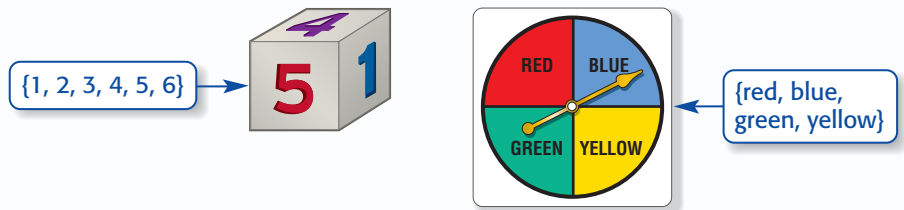
**WHEN** am I ever going to use this?

**MOVIES** A movie theater's concession stand sign is shown.

1. List the possible ways to choose a soft drink, a popcorn, and a candy.
2. How many different ways are possible?



The set of all possible outcomes is called the **sample space**. The list you made above is the sample space of choices at the concession stand. The sample space for rolling a number cube and spinning the spinner are listed below.

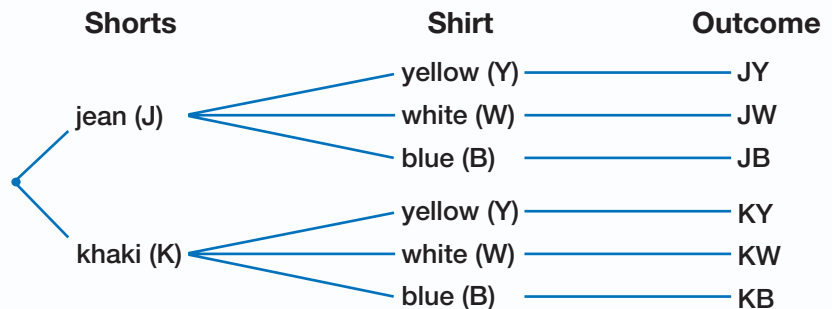


A tree diagram can also be used to show a sample space. When you make a tree diagram, you have an organized list of outcomes. A **tree diagram** is a diagram that shows all possible outcomes of an event.

### EXAMPLE Find a Sample Space

1. How many outfits are possible from a choice of jean or khaki shorts and a choice of a yellow, white, or blue shirt?

Use a tree diagram. List each choice for shorts. Then pair each choice for shorts with each choice for a shirt.



There are six possible outfits.

### READING Math

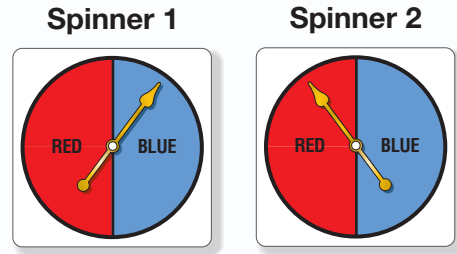
**Outcomes** The outcome JY means jean shorts and a yellow shirt.



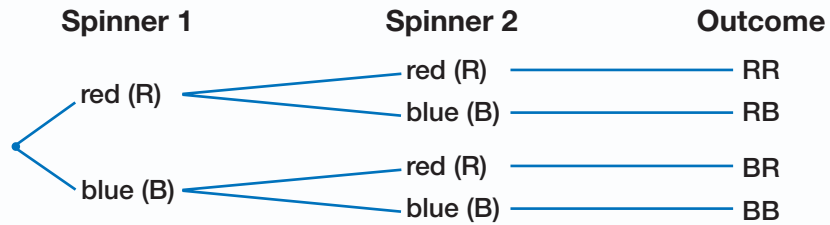
When you know the number of outcomes, you can easily find the probability that an event will occur.

**EXAMPLE** Use a Tree Diagram to Find Probability

- 1 Lexi spins two spinners. What is the probability of spinning red on the first spinner and blue on the second spinner?



Use a tree diagram to find all of the possible outcomes.



One outcome has red and then blue. There are four possible outcomes. So,  $P(\text{red, blue}) = \frac{1}{4}$ , 0.25, or 25%.

**Your Turn**

- Use a tree diagram to find how many different words can be made using the words *quick*, *slow*, and *sad* and the suffixes *-ness*, *-er*, and *-ly*.
- A penny is tossed, and a number cube is rolled. Use a tree diagram to find the probability of getting heads and a 5.

You can also make a list of possible outcomes.

**EXAMPLE** Use a List to Find Sample Space

- 1 **PETS** The names of three bulldog puppies are shown. In how many different ways can a person choose two of the three puppies? List all of the ways two puppies can be chosen.

Puppies' Names
Alex
Bailey
Chester

AB AC BA BC CA CB

From the list count only the different arrangements. In this case, AB is the same as BA.

AB AC BC

There are 3 ways to choose two of the puppies.

**Your Turn**

- How many different ways can a person choose three out of four puppies?

**STUDY TIP**

**Combinations**  
 Example 3 is an example of a *combination*. A combination is an arrangement, or listing, of objects in which order is *not* important.

## Skill and Concept Check

- Writing Math** Define *sample space* in your own words.
- OPEN ENDED** Give an example of a situation that has 8 outcomes.

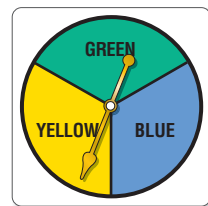
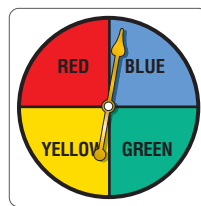
### GUIDED PRACTICE

Draw a tree diagram to show the sample space for each situation. Then tell how many outcomes are possible.

- hamburger or cheeseburger with a soft drink, water, or juice
- small, medium, large, or extra-large shirt in blue, white, or black

For Exercises 5 and 6, use the spinners shown. Each spinner is spun once.

- How many outcomes are possible?
- What is  $P(\text{blue, green})$  in that order?



## Practice and Applications

Draw a tree diagram to show the sample space for each situation. Then tell how many outcomes are possible.

- apple, peach, or cherry pie with milk, juice, or tea
- nylon, leather, or mesh backpack in red, blue, gold, or black
- roll two number cubes
- toss a dime, quarter, and penny

### HOMWORK HELP

For Exercises	See Examples
7–10	1
11–18	2
19–20	3

Extra Practice  
See pages 616, 634.

For Exercises 11–13, a coin is tossed, and a letter is chosen from the bag shown.

- How many outcomes are possible?
- Find  $P(\text{heads, E})$ .
- What is the probability of tails and P, Z, or M?



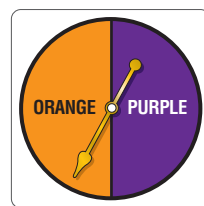
**SCHOOL** For Exercises 14 and 15, use the following information.

A science quiz has one multiple-choice question with answer choices A, B, and C, and two true/false questions.

- Draw a tree diagram that shows all of the ways a student can answer the questions.
- Find the probability of answering all three questions correctly by guessing.

For Exercises 16–18, a coin is tossed, the spinner shown is spun, and a number cube is rolled.

- How many outcomes are possible?
- What is  $P(\text{heads, purple, 5})$ ?
- Find  $P(\text{tails, orange, less than 4})$ .



19. **GAMES** How many ways can 2 video games be chosen from 5 video games?
20. **BOOKS** How many ways can 3 books be selected from 5 books?

**CRITICAL THINKING** For Exercises 21 and 22, use the following information.

One of the bags shown is selected without looking, and then one marble is selected from the bag without looking.



21. Draw a tree diagram showing all of the outcomes.
22. Is each outcome equally likely? Explain.

**EXTENDING THE LESSON**

A permutation is another way to show a sample space. A *permutation* is an arrangement or listing where order is important.

**Example**

How many ways can Katie, Jose, and Tara finish a race?

List the ways in a table as shown. There are 6 ways three people can finish a race.

1 <sup>st</sup> Place	2 <sup>nd</sup> Place	3 <sup>rd</sup> Place
Katie	Jose	Tara
Katie	Tara	Jose
Jose	Katie	Tara
Jose	Tara	Katie
Tara	Jose	Katie
Tara	Katie	Jose

Order is important because Katie, Jose, Tara is not the same as Jose, Tara, Katie.

Make a list to find the number of outcomes for each situation.

23. How many ways can Maria, Ryan, Liana, and Kurtis serve as president, vice-president, secretary, and treasurer?
24. How many ways can an editor and a reporter be chosen for their school paper from Jenna, Rico, Maralan, Ron, and Venessa?

**Spiral Review with Standardized Test Practice**

25. **MULTIPLE CHOICE** The menu for Wedgewood Pizza is shown. How many different one-topping pizzas can a customer order?

- (A) 4                      (B) 8                      (C) 16                      (D) 20

Wedgewood Pizza	
Size	Toppings
8-inch	cheese
10-inch	pepperoni
12-inch	sausage
14-inch	mushroom

26. **SHORT RESPONSE** How many different ways can you choose 2 of 4 different donuts?

A bag contains 5 red marbles, 6 green marbles, and 4 blue marbles. One marble is selected at random. Find each probability. (Lesson 11-1)

27.  $P(\text{green})$                       28.  $P(\text{green or blue})$                       29.  $P(\text{red or blue})$
30. Estimate 84% of 24. (Lesson 10-8)

**GETTING READY FOR THE NEXT LESSON**

**PREREQUISITE SKILL** Solve each proportion. (Lesson 8-2)

31.  $\frac{k}{9} = \frac{10}{45}$                       32.  $\frac{18}{72} = \frac{m}{24}$                       33.  $\frac{5}{c} = \frac{30}{96}$                       34.  $\frac{15}{35} = \frac{3}{d}$

**What You'll LEARN**

Determine whether a group is biased.

**Bias****INVESTIGATE** *Work in three large groups.*

Have you ever heard a listener win a contest on the radio? The disc jockey usually asks the listener "What's your favorite radio station?" When asked this question, the winner is usually **biased** because he or she favors the station that is awarding them a prize. In this lab, you will investigate bias.

**STEP 1**

Each group chooses one of the questions below.

**Question 1:** Should the amount of time between classes be lengthened since classrooms are far away?

**Question 2:** Our teacher will throw a pizza party if she is teacher of the year. Who is your favorite teacher?

**Question 3:** Many students in our school buy lunch. What is your favorite school lunch?

**STEP 2**

Each member of the group answers the question by writing his or her response on a sheet of paper.

**STEP 3**

Collect and then record the responses in a table.

**Writing Math**

*Work with a partner.*

1. **Compare** the responses of your group to the responses of the other groups. Which questions may result in bias?
2. **Describe** the ways in which the wording of these questions may have influenced your answers.
3. **Tell** how these questions can be rewritten so they do not result in answers that are biased.

Tell whether each of the following survey locations might result in bias.

Type of Survey	Survey Location
4. favorite hobby	model train store
5. favorite season	public library
6. favorite TV show	skating rink
7. favorite food	Mexican restaurant

# 11-3

## Statistics: Making Predictions

### HANDS-ON Mini Lab

#### What You'll LEARN

Predict the actions of a larger group using a sample.

#### NEW Vocabulary

survey  
population  
sample  
random

Work in three large groups.

In this activity, you will make a prediction about the number of left-handed or right-handed students in your school.

**STEP 1** Have one student in each group copy the table shown.

Left- or Right-Handed?	
Trait	Students
left-handed	
right-handed	

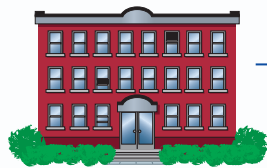
**STEP 2** Count the number of left-handed students and right-handed students in your group. Record the results.

**STEP 3** Predict the number of left-handed and right-handed students in your school.

**STEP 4** Combine your results with the other groups in your class. Make a class prediction.

1. When working in a group, how did your group predict the number of left-handed and right-handed students in your school?
2. Compare your group's prediction with the class prediction. Which do you think is more accurate? Explain.

A **survey** is a method of collecting information. The group being studied is the **population**. Sometimes, the population is very large. To save time and money, part of the group, called a **sample**, is surveyed.



The entire group of students in a school is an example of a population.

A classroom is a subset of the entire school.



The group of students in one classroom of a school is a sample of the population.

#### STUDY TIP

**Surveys** A survey has questions that require a response. The most common types of surveys are interviews, telephone surveys, or mailed surveys.

A good sample is:

- selected at **random**, or without preference,
- representative of the population, and
- large enough to provide accurate data.

**EXAMPLE****Determine a Good Sample**

1 Every ninth person entering a grocery store one day is asked to state whether they support the proposed school tax for their school district. Determine whether the sample is a good sample.

- Asking every ninth person ensures a random survey.
- The sample should be representative of the larger population; that is, every person living in the school's district.
- The sample is large enough to provide accurate information.

So, this sample is a good sample.

**Your Turn**

- a. One hundred people eating at an Italian restaurant are surveyed to name their favorite type of restaurant. Is this a good sample? Explain.

You can use the results of a survey to predict or estimate the actions of a larger group.

**EXAMPLES****Make Predictions Using Proportions**

**PIZZA** Lorenzo asked every tenth student who walked into school to name their favorite pizza topping.

Favorite Pizza Topping	
Topping	Students
pepperoni	18
cheese	9
sausage	3
mushroom	2



1 What is the probability that a student will prefer pepperoni pizza?

$$P(\text{pepperoni}) = \frac{\text{number of students that like pepperoni}}{\text{number of students surveyed}}$$

$$= \frac{18}{32}$$

So,  $P(\text{pepperoni})$  is  $\frac{18}{32}$ , 0.5625, or about 56%.

1 There are 384 students at the school Lorenzo attends. Predict how many students prefer pepperoni pizza.

Use a proportion.

Let  $s$  = students who prefer pepperoni.

$$\frac{18}{32} = \frac{s}{384}$$

Write the proportion.

$$18 \times 384 = 32 \times s$$

Write the cross products.

$$6,912 = 32s$$

Multiply.

$$\frac{6,912}{32} = \frac{32s}{32}$$

Divide each side by 32.

$$216 = s$$

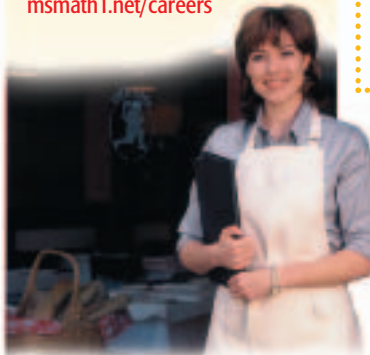
Of the 384 students, about 216 will prefer pepperoni pizza.

**REAL-LIFE CAREERS****How Does a Restaurant Manager Use Math?**

A restaurant manager uses math when he or she estimates food consumption, places orders with suppliers, and schedules the delivery of fresh food and beverages.

**Research**

For information about a career as a restaurant manager, visit: [msmath1.net/careers](http://msmath1.net/careers)





## Skill and Concept Check

- Writing Math** Tell how you would choose a random sample for a survey to find out how many students at your school ride the bus.
- FIND THE ERROR** Raheem and Elena are deciding on the best place to conduct a favorite movie star survey. Who is correct? Explain.

*Raheem*  
Ask people standing in a line waiting to see a movie.

*Elena*  
Ask people shopping at a local shopping center.

## GUIDED PRACTICE

Determine whether the following sample is a good sample. Explain.

- Every third student entering the school cafeteria is asked to name his or her favorite lunch food.

**FOOD** For Exercises 4 and 5, use the following information and the table shown.

Every tenth person entering a concert is asked to name his or her favorite milk shake flavor.

- Find the probability that any person attending the concert prefers chocolate milk shakes.
- Predict how many people out of 620 would prefer chocolate milk shakes.

Favorite Milk Shake	
Milk Shake	People
vanilla	30
chocolate	15
strawberry	10
mint	5

## Practice and Applications

Determine whether each sample is a good sample. Explain.

- Fifty children at a park are asked whether they like to play indoors or outdoors.
- Every twentieth student leaving school is asked where the school should hold the year-end outing.

**SOC CER** For Exercises 8 and 9, use the following information.

In soccer, Isabelle scored 4 goals in her last 10 attempts.

- Find the probability of Isabelle scoring a goal on her next attempt.
- Suppose Isabelle attempts to score 20 goals. About how many goals will she make?

**MUSIC** For Exercises 10–13, use the table at the right to predict the number of students out of 450 that would prefer each type of music.

- rock
- country
- alternative
- pop

Favorite Type of Music	
Music	Students
pop	9
rock	5
country	2
rap	5
alternative	4

## HOMWORK HELP

For Exercises	See Examples
6–7	1
8	2
9–15	3

**Extra Practice**  
See pages 617, 634.

**Kids care about the community**

Do they volunteer?



Source: ZOOM and Applied Research &amp; Consulting LLC

By Mary M. Kershaw and Robert W. Ahrens, USA TODAY

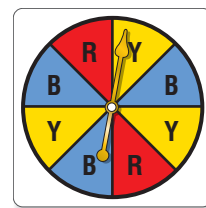
**VOLUNTEERING** For Exercises 14 and 15, use the graph at the right.

- There were about 300,000 kids aged 10–14 living in Colorado in 2000. Predict the number of kids that volunteered.
- In 2000, there were about 75,000 kids aged 10–14 living in Hawaii. Make a prediction as to the number of kids in this age group that did not volunteer.



**Data Update** Find an estimate for the number of kids aged 10–14 currently living in your state. Predict the number of these kids that volunteer. Visit [msmath1.net/data\\_update](http://msmath1.net/data_update) to learn more.

- CRITICAL THINKING** Use the spinner shown at the right. If the spinner is spun 400 times, about how many times will the spinner stop on red or yellow?



## Spiral Review with Standardized Test Practice

- MULTIPLE CHOICE** Students in a classroom were asked to name their favorite pet. The results are shown. If there are 248 students in the school, how many will prefer cats?
 

Ⓐ 82      Ⓑ 74      Ⓒ 62      Ⓓ 25
- MULTIPLE CHOICE** Which sample is *not* a good sample?
 

Ⓕ survey: favorite flower; location: mall

Ⓖ survey: favorite hobby; location: model train store

Ⓗ survey: favorite holiday; location: fast-food restaurant

Ⓙ survey: favorite sport; location: library
- How many ways can a person choose 3 videos from a stack of 6 videos? (Lesson 11-2)

Pet	Students
dog	12
cat	9
hamster	4
turtle	4
rabbit	2
guinea pig	5

Sarah randomly turns to a page in a 12-month calendar. Find each probability. (Lesson 11-1)

20.  $P(\text{April or May})$

21.  $P(\text{not June})$

22.  $P(\text{begins with a J})$

### GETTING READY FOR THE NEXT LESSON

**PREREQUISITE SKILL** Write each fraction in simplest form. (Lesson 5-2)

23.  $\frac{4}{16}$

24.  $\frac{12}{36}$

25.  $\frac{8}{20}$

26.  $\frac{9}{24}$



## Vocabulary and Concepts

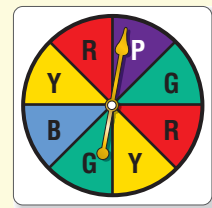
1. **Explain** how a ratio is used to find the probability of an event.  
(Lesson 11-1)
2. In your own words, **describe** *sample*. (Lesson 11-3)

## Skills and Applications

The spinner is spun once. Find each probability. Write each answer as a fraction, a decimal, and a percent.

(Lesson 11-1)

3.  $P(\text{red})$
4.  $P(\text{blue or red})$
5.  $P(\text{orange})$
6.  $P(\text{not yellow})$
7. Draw a tree diagram to show the sample space for a choice of chicken, beef, or fish and rice or potatoes for dinner. (Lesson 11-2)
8. **PETS** How many different ways can a person choose 2 hamsters from a group of 4 hamsters? (Lesson 11-2)
9. How many outcomes are possible if a coin is tossed once and a number cube is rolled once? (Lesson 11-2)
10. Every tenth person leaving a grocery store is asked his or her favorite color. Determine whether the sample is a good sample. Explain. (Lesson 11-3)



## Standardized Test Practice

11. **MULTIPLE CHOICE** There is a 25% chance that tomorrow's baseball game will be cancelled due to bad weather. What is the probability that the baseball game will *not* be cancelled?  
(Lesson 11-1)
12. **SHORT RESPONSE** Use the results shown to predict the number of people out of 300 that would prefer vanilla ice cream. (Lesson 11-3)

- (A) 85%                      (B) 75%
- (C) 65%                      (D) 55%

Favorite Ice Cream	
Flavor	Amount
vanilla	9
chocolate	8
strawberry	3

# The Game Zone

A Place To Practice Your Math Skills

Math Skill

Finding Probabilities

## Match 'Em Up

### ● GET READY!

**Players:** two

**Materials:** 16 index cards

### ● GET SET!

- Make a set of 8 probability cards using the numbers below.

$\frac{1}{6}$   $\frac{1}{2}$   $0$   $\frac{1}{3}$   $\frac{5}{6}$   $\frac{2}{3}$   $1$   $\frac{1}{2}$

- Make a set of 8 event cards describing the results of rolling a number cube.

6	3, 4, or 5	7	less than 6
composite	1 or prime	odd	greater than 0

### ● GO!

- Mix up each set of cards and place the cards facedown as shown.
- Player 1 turns over an event card and a probability card. If the number corresponds to the probability that the event occurs, the player removes the cards, a point is scored, and the player turns over two more cards.
- If the fraction does not correspond to the event, the player turns the cards facedown, no points are scored, and it is the next player's turn.
- Take turns until all cards are matched.
- **Who Wins?** The player with more points wins.

Event Cards



Probability Cards



# Geometry: Probability and Area

## What You'll LEARN

Find probability using area models.

### HANDS-ON Mini Lab

Work with a partner.

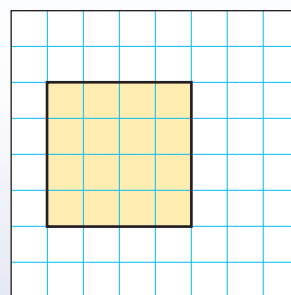
Let's investigate the relationship between area and probability.

- Copy the squares on grid paper and shade them as shown.
- Randomly drop 50 pieces of rice onto the squares, from about 8 inches above.
- Record the number of pieces of rice that land in the blue region.

1. Find  $P$ (a piece of rice lands in blue region).
2. How does this ratio compare to the ratio  $\frac{\text{area of blue region}}{\text{area of target}}$ ? Explain.

### Materials

- centimeter grid paper
- rice
- colored pencils



With a very large sample, the experimental probability should be close to the theoretical probability. The probability can be expressed as the ratio of the areas.

### Noteables™

### Key Concept: Probability and Area

**Words** The probability of landing in a specific region of a target is the ratio of the area of the specific region to the area of the target.

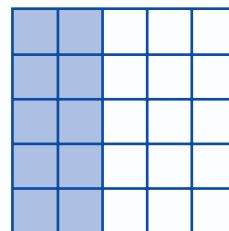
**Symbols**  $P(\text{specific region}) = \frac{\text{area of specific region}}{\text{area of the target}}$

### EXAMPLE Find Probability Using Area Models

- 1 Find the probability that a dart thrown randomly will land in the shaded region.

$$\begin{aligned} P(\text{shaded region}) &= \frac{\text{area of shaded region}}{\text{area of the target}} \\ &= \frac{10}{25} \text{ or } \frac{2}{5} \end{aligned}$$

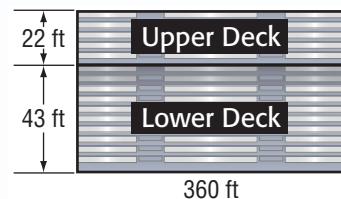
So, the probability is  $\frac{2}{5}$ , 0.4, or 40%.



## EXAMPLES

## Use Probability to Make Predictions

- 1 CHEERLEADING** A cheerleading squad plans to throw T-shirts into the stands using a sling shot. Find the probability that a T-shirt will land in the upper deck of the stands. Assume it is equally likely for a shirt to land anywhere in the stands.



$$P(\text{upper deck}) = \frac{\text{area of upper deck}}{\text{area of the stands}}$$

**Area of Upper Deck**

$$\begin{aligned} \ell \times w &= 360 \times 22 \\ &= 7,920 \text{ sq ft} \end{aligned}$$

**Area of Stands**

$$\begin{aligned} \ell \times w &= 360 \times 65 \\ &= 23,400 \text{ sq ft} \end{aligned}$$

The upper and the lower deck make up the stands.

$$\begin{aligned} P(\text{upper deck}) &= \frac{\text{area of upper deck}}{\text{area of the stands}} \\ &= \frac{7,920}{23,400} \\ &\approx \frac{1}{3} \end{aligned}$$

So, the probability that a T-shirt will land in the upper deck of the stands is about  $\frac{1}{3}$ ,  $0.33\bar{3}$ , or  $33.\bar{3}\%$ .

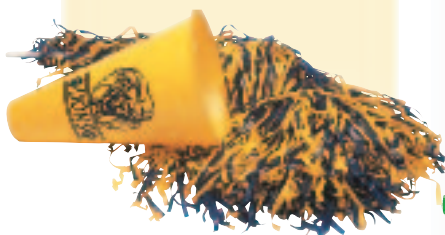
### STUDY TIP

**Look Back** To review area of rectangles, see Lesson 1-8.

### REAL-LIFE MATH

**CHEERLEADING** Eighty percent of the schools in the U.S. have cheerleading squads. The most popular sport for cheerleading is football.

Source: www.about.com



- 2 CHEERLEADING** Predict how many times a T-shirt will land in the upper deck of the stands if the cheerleaders throw 15 T-shirts.

Write a proportion that compares the number of T-shirts landing in the upper deck to the number of T-shirts thrown. Let  $n$  = the number of T-shirts landing in the upper deck.

$$\frac{n}{15} = \frac{1}{3} \quad \begin{array}{l} \leftarrow \text{T-shirts landing in upper deck} \\ \leftarrow \text{T-shirts thrown} \end{array}$$

$$n \times 3 = 15 \times 1 \quad \text{Write the cross products.}$$

$$3n = 15 \quad \text{Multiply.}$$

$$\frac{3n}{3} = \frac{15}{3} \quad \text{Divide each side by 3.}$$

$$n = 5$$

About 5 T-shirts will land in the upper deck.

### Your Turn

- Refer to the diagram in Example 2. Find the probability that a T-shirt will land in the lower deck of the stands.
- Predict the number of T-shirts that will land in the lower deck if 36 T-shirts are thrown.
- Predict the number of T-shirts that will land in the lower deck if 90 T-shirts are thrown.

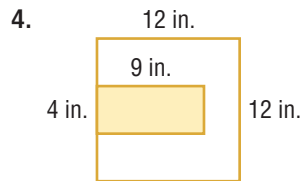
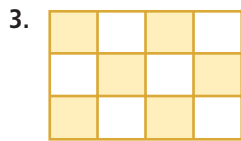


## Skill and Concept Check

- OPEN ENDED** Draw a dartboard in which the probability of a dart landing in the shaded area is 60%.
- Writing Math** Draw a model in which the probability of a dart landing in the shaded region is 25%. Then explain how to change the model so that the probability of a dart landing in the shaded region is 50%.

## GUIDED PRACTICE

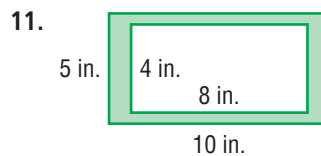
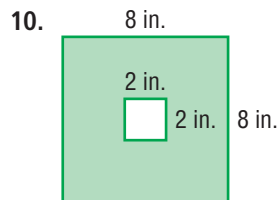
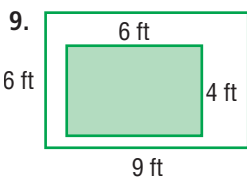
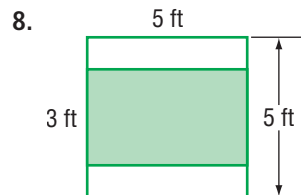
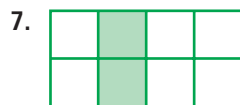
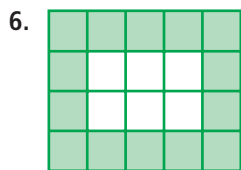
Find the probability that a randomly thrown dart will land in the shaded region of each dartboard.



5. If a dart is randomly thrown 640 times at the dartboard in Exercise 4, about how many times will the dart land in the shaded region?

## Practice and Applications

Find the probability that a randomly thrown dart will land in the shaded region of each dartboard.



### HOMEWORK HELP

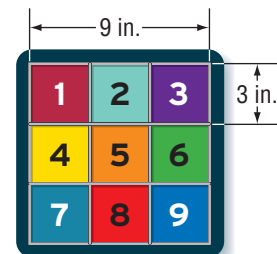
For Exercises	See Examples
6–11	1
12–17	2, 3

**Extra Practice**  
See pages 617, 634.

**GAMES** For Exercises 12–14, use the following information and the dartboard shown.

To win a prize, a dart must land on an even number. It is equally likely that the dart will land anywhere on the dartboard.

- What is the probability of winning a prize?
- Predict how many times a dart will land on an even number if it is thrown 9 times.
- MULTI STEP** To play the game, it costs \$2 for 3 darts. About how much money would you have to spend in order to win 5 prizes?

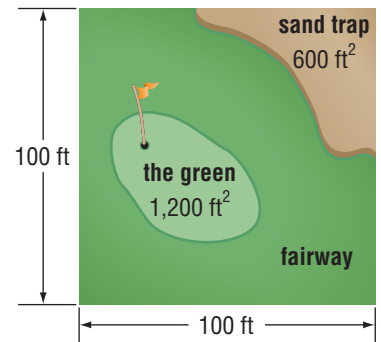


15. If a dart is randomly thrown 320 times at the dartboard in Exercise 10, about how many times will the dart land in the shaded region?

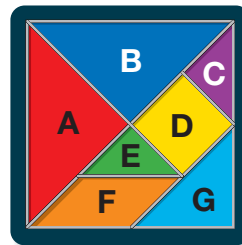
**GOLF** For Exercises 16 and 17, use the following information and the diagram at the right that shows part of one hole on a golf course.

Suppose a golfer tees off and it is equally likely that the ball lands somewhere in the area of the course shown.

16. What is the probability that the ball lands in the sand trap?
17. If the golfer tees off from this hole 50 times, how many times can he expect the ball to land somewhere on the green?



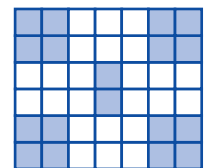
18. **CRITICAL THINKING** Predict the number of darts that will land in each region of the dartboard shown if a dart is randomly thrown 200 times.



## Spiral Review with Standardized Test Practice

19. **MULTIPLE CHOICE** Find the probability that a randomly thrown dart will land in the shaded region of the dartboard shown.

- Ⓐ  $\frac{2}{7}$       Ⓑ  $\frac{2}{5}$       Ⓒ  $\frac{3}{7}$       Ⓓ  $\frac{3}{4}$



20. **MULTIPLE CHOICE** If 100 darts are thrown at the dartboard above, about how many would you expect to land in the shaded region?

- Ⓕ 40      Ⓖ 43      Ⓗ 45      Ⓘ 46

21. Every tenth person entering the main entrance of a Yankees' baseball game is asked to name their favorite baseball player. Determine whether this is a good sample. Explain. (Lesson 11-3)

22. **LANGUAGE** How many ways can a teacher choose 3 vocabulary words to put on a quiz from 4 different vocabulary words? (Lesson 11-2)

Find the percent of each number. (Lesson 10-7)

23. 30% of 60

24. 8% of 12

25. 110% of 150

### GETTING READY FOR THE NEXT LESSON

**PREREQUISITE SKILL** Multiply. Write in simplest form. (Lesson 7-2)

26.  $\frac{1}{5} \times \frac{3}{4}$

27.  $\frac{2}{3} \times \frac{3}{7}$

28.  $\frac{8}{15} \times \frac{3}{4}$

29.  $\frac{4}{7} \times \frac{14}{15}$





# 11-5a

## Problem-Solving Strategy

A Preview of Lesson 11-5

### What You'll LEARN

Solve problems by making a table.

### Make a Table

Hey Mario, my cat is going to have kittens! I'm hoping for three kittens, so I can keep one and give two to my cousins.

If your cat does have three kittens, what's the probability that she will have one female and two males? Let's **use a table** to find out.



#### Explore

Each kitten will be either a male or a female. We need to find the probability that one kitten is female and the other two kittens are male.

#### Plan

Make a table to list all of the possibilities to find the probability.

#### Solve

	1 <sup>st</sup> Kitten		2 <sup>nd</sup> Kitten		3 <sup>rd</sup> Kitten		Outcomes
	F	M	F	M	F	M	
		X		X		X	MMM
		X		X	X		MMF
		X	X			X	MFM
		X	X		X		MFF
X	X			X		X	FMM
X				X	X		FMF
X			X			X	FFM
X			X		X		FFF

There are 3 ways a female, male, and male can be born. There are 8 possible outcomes. So, the probability is  $\frac{3}{8}$ , or 0.375, or 37.5%.

#### Examine

If you draw a tree diagram to find the possible outcomes, you will find that the answer is correct.

### Analyze the Strategy

1. **Explain** when to use the make a table strategy to solve a problem.
2. **Tell** the advantages of organizing information in a table. What are the disadvantages?
3. **Write** a problem that can be solved using the make a table strategy.

## Apply the Strategy

Solve. Use the make a table strategy.

4. **CHILDREN** Find the probability that in a family with three children there is one boy and two girls.
5. **TESTS** A list of test scores is shown.  
 68 77 99 86 73 75 100  
 86 70 97 93 80 91 72  
 85 98 79 77 65 89 71  
 How many more students scored 71 to 80 than 91 to 100?

6. **SCHOOL** The birth months of the students in Miss Miller's geography class are shown below. Make a frequency table of the data. How many more students were born in June than in August?

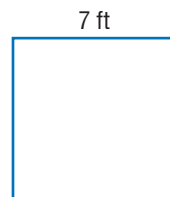
Birth Months		
June	July	April
March	July	June
October	May	August
June	April	October
May	October	April
September	December	January

## Mixed Problem Solving

Solve. Use any strategy.

7. **MONEY** Julissa purchased her school uniforms for \$135. This was the price after a 15% discount. What was the original price of her uniforms?

**GEOMETRY** For Exercises 8 and 9, use the square shown at the right.



8. Find the length of each side of the square.
9. What is the perimeter of the square?
10. **GAMES** Sara tosses a beanbag onto an alphabet board. It is equally likely that the bag will land on any letter. Find the probability that the beanbag will land on one of the letters in her name.
11. **SCHOOL** Of the 150 students at Lincoln Middle School, 55 are in the orchestra, 75 are in marching band, and 25 are in both orchestra and marching band. How many students are in neither orchestra nor marching band?
12. **MONEY** Tetuso bought a clock radio for \$9 less than the regular price. If he paid \$32, what was the regular price?

13. **ROLLER COASTERS** The list shows how many roller coasters 20 kids rode at an amusement park.

5 10 0 12 8 7 2 6 4 1  
 0 6 3 11 5 9 13 8 14 3

Make a frequency table to find how many more kids ride roller coasters 5 to 9 times than 10 to 14 times.

14. **MONEY** Luke collected \$2 from each student to buy a gift for their teacher. If 27 people contributed, how much money was collected?

15. **STANDARDIZED TEST PRACTICE**

Fabric that costs \$6.59 per yard is on sale for 20% off per yard. Abigail needs to purchase  $5\frac{3}{8}$  yards of the fabric. Which expression shows the amount of change  $c$  she should receive from a \$50 bill?

(A)  $c = 50 - (6.59 + 0.20 + 5\frac{3}{8})$

(B)  $c = 50 - (6.59)(0.80) - 5\frac{3}{8}$

(C)  $c = 50 - (6.59)(0.80) (5\frac{3}{8})$

(D)  $c = 50 + (6.59)(0.80) (5\frac{3}{8})$

# 11-5

## Probability of Independent Events

### What You'll LEARN

Find the probability of independent events.

### NEW Vocabulary

independent events

### HANDS-ON Mini Lab

Work with a partner.

Make a tree diagram that shows the sample space for rolling a number cube and choosing a marble from the bag.



### Materials

- number cube
- bag
- 5 marbles

1. How many outcomes are in the sample space?
2. What is the probability of rolling a 5 on the number cube?
3. Find the probability of selecting a yellow marble.
4. Use the tree diagram to find  $P(5 \text{ and yellow})$ .
5. Describe the relationship between  $P(5)$ ,  $P(\text{yellow})$ , and  $P(5 \text{ and yellow})$ .

In the Mini Lab, the outcome of rolling the number cube does not affect the outcome of choosing a marble. Two or more events in which the outcome of one event does not affect the outcome of the other event are **independent events**.

### Noteables™

### Key Concept: Probability of Independent Events

The probability of two independent events is found by multiplying the probability of the first event by the probability of the second event.

### EXAMPLE

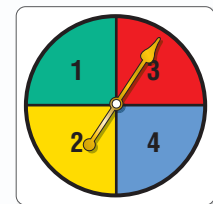
### Find Probability of Independent Events

- 1 A coin is tossed, and the spinner shown is spun. Find the probability of tossing heads and spinning a 3.

$$P(\text{heads}) = \frac{1}{2} \quad P(3) = \frac{1}{4}$$

$$P(\text{heads and } 3) = \frac{1}{2} \times \frac{1}{4} \text{ or } \frac{1}{8}$$

So, the probability is  $\frac{1}{8}$ , 0.125, or 12.5%.



- 2 **Your Turn** Find the probability of each event.

- a.  $P(\text{tails and even})$
- b.  $P(\text{heads and less than } 4)$

### STUDY TIP

**Look Back** To review multiplying fractions, see Lesson 7-2.



### EXAMPLE

## Find Probability of Independent Events

**1 GRID-IN TEST ITEM** Amanda placed 2 red marbles and 6 yellow marbles into a bag. She selected 1 marble without looking, replaced it, and then selected a second marble. Find the probability that each marble selected was *not* yellow.

**Read the Test Item** To find the probability, find  $P(\text{not yellow and not yellow})$ .

### Solve the Test Item

First marble:  $P(\text{not yellow}) = \frac{2}{8}$  or  $\frac{1}{4}$

Second marble:  $P(\text{not yellow}) = \frac{2}{8}$  or  $\frac{1}{4}$

So,  $P(\text{not yellow and not yellow})$  is  $\frac{1}{4} \times \frac{1}{4}$  or  $\frac{1}{16}$ .

### Fill in the Grid

	1	/	1	6
	⊘	●	⊘	
⊘	⊘	⊘	⊘	⊘
0	0	0	0	0
1	●	1	●	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	●
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

### Test-Taking Tip

#### Read the Test Item

When reading the question, always look for more than one way to solve the problem. For example, the probability question in Example 2 can also be solved by finding  $P(\text{red and red})$ .

## Skill and Concept Check

- Writing Math** Explain how to find the probability of two independent events.
- OPEN ENDED** Give an example of two independent events.
- Which One Doesn't Belong?** Identify the situation that is *not* a pair of independent events. Explain your reasoning.

tossing two coins

choosing one marble from a bag of marbles without replacing it

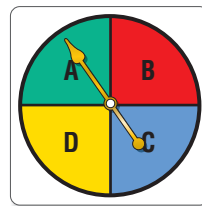
rolling two number cubes

spinning two spinners

### GUIDED PRACTICE

One marble is selected from the bag without looking, and the spinner is spun. Find each probability.

- $P(\text{D and blue})$
- $P(\text{B and yellow})$
- $P(\text{consonant and green})$



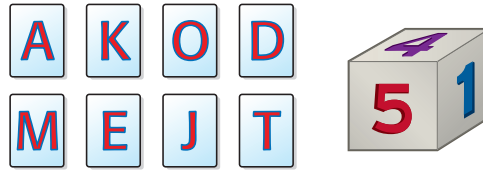
- PETS** What is the probability that in a litter of 2 puppies the first puppy born is female and the second puppy born is male?
- PENS** A desk drawer contains 8 blue pens and 4 red pens. Suppose one pen is selected without looking, replaced, and another pen is selected. What is the probability that the first pen selected is blue and the second pen is red?



## Practice and Applications

A letter card is chosen, and a number cube is rolled. Find the probability of each event.

9.  $P(A \text{ and } 5)$
10.  $P(K \text{ and even})$
11.  $P(J \text{ and less than } 3)$
12.  $P(\text{vowel and } 7)$
13.  $P(\text{consonant and odd})$
14.  $P(M \text{ or } T \text{ and greater than } 3)$



### HOMEWORK HELP

For Exercises	See Examples
9–15	1
17–26	2

Extra Practice  
See pages 617, 634.

**SPORTS** For Exercises 15 and 16, use the following information.

The probability that the Jets beat the Zips is 0.4. The probability that the Jets beat the Cats is 0.6.

15. What is the probability that the Jets beat both the Zips and the Cats?
16. Explain whether the Jets beating both the Zips and the Cats is less likely or more likely to happen than the Jets beating either the Zips or the Cats.

One marble is chosen from the bag shown without looking, replaced, and another marble chosen. Find each probability.

17.  $P(\text{green and red})$
18.  $P(\text{green and orange})$
19.  $P(\text{purple and red})$
20.  $P(\text{orange and blue})$
21.  $P(\text{green and purple})$
22.  $P(\text{purple and purple})$



23. **LIFE SCIENCE** The table lists the items Scott and Tyrone collected while on a nature walk.

	Name	Stones	Pinecone	Acorn
	Scott	12	12	16
	Tyrone	6	8	10

Each person reaches into his bag and randomly selects an object. Find the probability that Scott chooses an acorn and Tyrone chooses a stone.

24. **SCHOOL** A quiz has one true/false question and one multiple-choice question with possible answer choices A, B, C, and D. If you guess each answer, what is the probability of answering both questions correctly?

**CANDY** For Exercises 25 and 26, use the graph at the right. It shows the colors of candy that come in a bag of chocolate candies. Suppose one candy is chosen without looking, replaced, and another candy chosen.

25. Find  $P(\text{red and not green})$ .
26. What is  $P(\text{blue or green and not yellow})$ ?

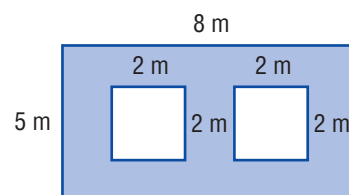


27. **RESEARCH** Use the Internet or another source to find one example of probability being used in everyday life. Write a probability question and then solve your question.
28. Give a counterexample for the following statement.  
*If the probability of event A is less than  $\frac{3}{4}$  and the probability of event B is less than  $\frac{1}{2}$ , then the probability of A and B is less than  $\frac{1}{4}$ .*
29. **CRITICAL THINKING** Two spinners are spun. The probability of landing on red on both spinners is 0.2. The probability of landing on red on the second spinner is 0.5. What is the probability of landing on red on the first spinner?

**Spiral Review with Standardized Test Practice**

30. **MULTIPLE CHOICE** A coin is tossed and a card is chosen from a set of ten cards labeled 1–10. Find  $P(\text{tails, prime})$ .  
 Ⓐ  $\frac{1}{10}$       Ⓑ  $\frac{1}{5}$       Ⓒ  $\frac{3}{10}$       Ⓓ  $\frac{2}{5}$
31. **MULTIPLE CHOICE** A yellow and a blue number cube are rolled. Find the probability of rolling a 6 on the yellow number cube and a number less than 5 on the blue number cube.  
 Ⓕ  $\frac{1}{18}$       Ⓖ  $\frac{1}{9}$       Ⓗ  $\frac{1}{6}$       Ⓘ  $\frac{2}{3}$

32. It is equally likely that a thrown dart will land anywhere on the dartboard shown. Find the probability of a randomly thrown dart landing in the shaded region.  
 (Lesson 11-4)



33. **SOCCER** Ian scores 6 goals in every 10 attempts. About how many goals will he score if he attempts 15 goals?  
 (Lesson 11-3)

**Subtract. Write in simplest form.** (Lesson 6-5)

34.  $5\frac{1}{6} - 3\frac{8}{9}$


35.  $8\frac{2}{3} - 3\frac{6}{7}$

36.  $6\frac{3}{5} - 4\frac{2}{3}$

37.  $9\frac{2}{3} - 5\frac{4}{5}$

**INTERDISCIPLINARY PROJECT**

**Take Me Out to the Ballgame**  
**Math and Sports** It's time to complete your project. Use the scale drawing you've created and the data you have gathered about your baseball teams to prepare a Web page or poster. Be sure to include a spreadsheet with your project.

 [msmath1.net/webquest](http://msmath1.net/webquest)

## Vocabulary and Concept Check

complementary events (p. 429)  
event (p. 428)  
independent events (p. 450)  
outcomes (p. 428)

population (p. 438)  
random (p. 438)  
sample (p. 438)  
sample space (p. 433)

survey (p. 438)  
theoretical probability (p. 428)  
tree diagram (p. 433)

Choose the letter of the term that best matches each phrase.

- the ratio of the number of ways an event can occur to the number of possible outcomes
- a specific outcome or type of outcome
- when one event occurring does not affect another event
- events that happen by chance
- a diagram used to show all of the possible outcomes
- the set of all possible outcomes
- a randomly selected group chosen for the purpose of collecting data

- tree diagram
- outcomes
- random events
- event
- sample
- probability
- independent events
- sample space

## Lesson-by-Lesson Exercises and Examples

## 11-1 Theoretical Probability (pp. 428–431)

One coin shown is chosen without looking. Find each probability. Write each answer as a fraction, a decimal, and a percent.



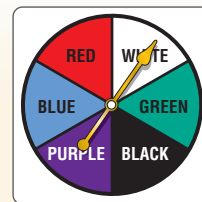
- $P(\text{nickel})$
- $P(\text{not dime})$
- $P(\text{quarter or penny})$
- $P(\text{nickel or dime})$

A number cube is rolled. Find each probability. Write each answer as a fraction, a decimal, and a percent.

- $P(5)$
- $P(\text{less than } 4)$
- $P(\text{odd})$
- $P(\text{at least } 5)$

## Example 1

The spinner shown is spun once. Find the probability of spinning blue.



There are six equally likely outcomes on the spinner. One of the six is blue.

$$\begin{aligned}
 P(\text{blue}) &= \frac{\text{number of ways to spin blue}}{\text{total number of possible outcomes}} \\
 &= \frac{1}{6}
 \end{aligned}$$

### 11-2 Outcomes (pp. 433–436)

Draw a tree diagram to show the sample space for each situation. Then tell how many outcomes are possible.

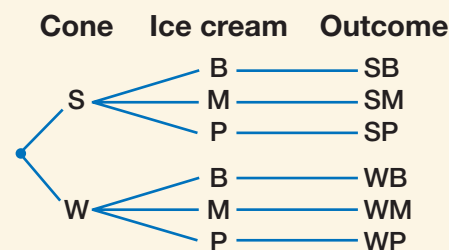
- a choice of black or blue jeans in tapered, straight, or baggy style
- a choice of soup or salad with beef, chicken, fish, or pasta
- a choice of going to a basketball game, an amusement park, or a concert on a Friday or a Saturday

A coin is tossed, and a number cube is rolled.

- How many outcomes are possible?
- Find  $P(\text{tails}, 2)$ .
- MARBLES** How many ways can 3 marbles be selected from a bag of 6 different marbles?

**Example 2** Suppose you have a choice of a sugar cone (S) or a waffle cone (W) and blueberry (B), mint (M), or peach (P) ice cream.

- How many ice cream cones are possible?



There are 6 possible ice cream cones.

- If you choose at random, find the probability of selecting a sugar cone with blueberry ice cream or a waffle cone with mint ice cream.

$$\begin{aligned} P(\text{blueberry/sugar or mint/waffle}) \\ = \frac{2}{6} \text{ or } \frac{1}{3} \end{aligned}$$

### 11-3 Statistics: Making Predictions (pp. 438–441)

**SCHOOL** For Exercises 22 and 23, use the following information.

Out of 40 students, 14 are interested in publishing a school newspaper.

- What is the probability that a student at this school would be interested in publishing a school newspaper?
- If there are 420 students, how many would you expect to be interested in publishing a school newspaper?
- Twenty residents of Florida are asked whether they prefer warm or cold weather. Determine whether the sample is a good sample. Explain.

**Example 3** If 12 out of 50 people surveyed prefer to watch TV after 11 P.M., how many people out of 1,000 would prefer to watch TV after 11 P.M.?

Let  $p$  represent the number of people who would prefer to watch TV after 11 P.M.

$$\frac{12}{50} = \frac{p}{1,000} \quad \text{Use a proportion.}$$

$$12 \times 1,000 = 50 \times p \quad \text{Find the cross products.}$$

$$12,000 = 50p \quad \text{Multiply.}$$

$$\frac{12,000}{50} = \frac{50p}{50} \quad \text{Divide.}$$

$$240 = p \quad \text{Simplify.}$$

Of the 1,000 people, 240 would prefer to watch TV after 11 P.M.



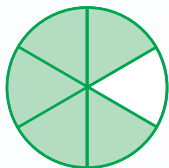
**Mixed Problem Solving**

 For mixed problem-solving practice,  
see page 634.

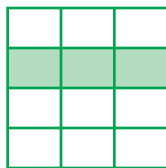
**11-4 Geometry: Probability and Area** (pp. 444–447)

Find the probability that a randomly thrown dart will land in the shaded region of each dartboard.

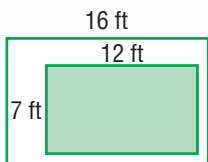
25.



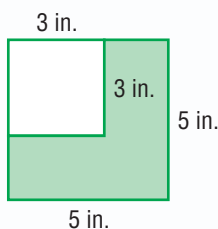
26.



27.

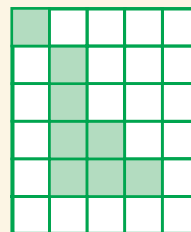


28.



29. Suppose you threw a dart 150 times at the dartboard in Exercise 28. How many times would you expect it to land in the shaded region?

**Example 4** The figure shown represents a dartboard. Find the probability that a randomly thrown dart lands in the shaded region.



$$\begin{aligned} P(\text{shaded region}) &= \frac{\text{area of shaded region}}{\text{area of target}} \\ &= \frac{8}{36} \text{ or } \frac{4}{18} \end{aligned}$$

So, the probability is  $\frac{4}{15}$ ,  $1.26\bar{6}$ , or  $26.\bar{6}\%$ .

**11-5 Probability of Independent Events** (pp. 450–453)

A coin is tossed and a number cube is rolled. Find the probability of each event.



30.  $P(\text{heads and } 4)$
  31.  $P(\text{tails and even})$
  32.  $P(\text{heads and } 5 \text{ or } 6)$
  33.  $P(\text{tails and } 2, 3, \text{ or } 4)$
  34.  $P(\text{heads and prime})$
35. **EARTH SCIENCE** The probability of rain on Saturday is 0.6. The probability of rain on Sunday is 0.3. What is the probability that it will rain on both days?

**Example 5** Two number cubes are rolled.

- a. Find  $P(\text{odd and } 4)$ .

$$P(\text{odd}) = \frac{1}{2} \quad P(4) = \frac{1}{6}$$

$$P(\text{odd and } 4) = \frac{1}{2} \times \frac{1}{6} \text{ or } \frac{1}{12}$$

So, the probability is  $\frac{1}{12}$ ,  $0.08\bar{3}$ , or  $8.\bar{3}\%$ .

- b. What is  $P(6 \text{ and less than } 5)$ ?

$$P(6) = \frac{1}{6} \quad P(\text{less than } 5) = \frac{2}{3}$$

$$P(6 \text{ and less than } 5) = \frac{1}{6} \times \frac{2}{3} \text{ or } \frac{1}{9}$$

So, the probability is  $\frac{1}{9}$ ,  $0.11\bar{1}$ , or  $11.\bar{1}\%$ .

## Practice Test

## Vocabulary and Concepts

- List three characteristics of a good sample.
- Define *independent events*.

## Skills and Applications

A set of 20 cards is numbered 1–20. One card is chosen without looking. Find each probability. Write as a fraction, a decimal, and a percent.

- $P(8)$
- $P(3 \text{ or } 10)$
- $P(\text{prime})$
- $P(\text{odd})$
- PENS** How many ways can 2 pens be chosen from 4 pens?

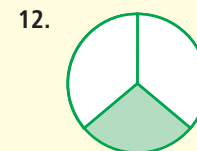
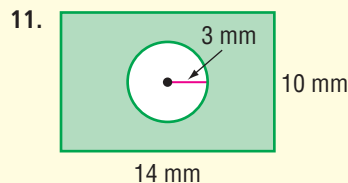
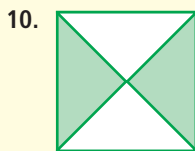
**MUSIC** For Exercises 8 and 9, use the table at the right and the following information.

Alonso asked every fourth sixth grade student who walked into a school dance to name their favorite sport.

- Find the probability a student prefers football.
- If there are 375 students in the sixth grade, how many can be expected to prefer football?

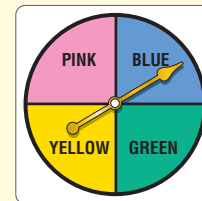
Favorite Sport	
Sport	Students
football	52
soccer	22
baseball	16
hockey	10

Find the probability that a randomly thrown dart will land in the shaded region of each dartboard.



A coin is tossed and the spinner shown is spun. Find the probability of each event.

- $P(\text{tails and blue})$
- $P(\text{heads and not green})$
- $P(\text{tails and pink, yellow, or green})$



## Standardized Test Practice

- MULTIPLE CHOICE** Which of the following is *not* a good sample?
  - survey: best movie; location: home improvement store
  - survey: worst song; location: park
  - survey: favorite dessert; location: ice cream shop
  - survey: least favorite school subject; location: school play



**PART 1** Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Find  $200 \times 2$ . (Prerequisite Skill, p. 590)

- (A) 4                      (B) 40  
(C) 100                    (D) 400

2. Which list orders 34.1, 33.9, 33.8, 34.2, 34.9 from least to greatest? (Lesson 3-2)

- (F) 34.1, 33.9, 34.2, 33.8, 34.9  
(G) 33.8, 33.9, 34.1, 34.2, 34.9  
(H) 34.1, 34.9, 34.2, 33.9, 33.8  
(I) 33.9, 33.8, 34.1, 34.2, 34.9

3. To the nearest tenth, how many times greater is the cost of a pancake breakfast at Le Café than at Pancakes & More?

Pancake Breakfast	
Restaurant	Cost
Skyview Chalet	\$3.60
Delightful Diner	\$4.10
Pancakes & More	\$2.20
Le Café	\$6.40

(Lesson 4-4)

- (A) 2.9                      (B) 3.1  
(C) 3.4                      (D) 4.2

4. What is 0.40 written as a fraction?

(Lesson 5-6)

- (F)  $\frac{4}{100}$                       (G)  $\frac{6}{30}$   
(H)  $\frac{2}{5}$                         (I)  $\frac{40}{10}$

5. What is the value of  $\frac{7}{12} \times \frac{4}{15}$ ? (Lesson 7-2)

- (A)  $\frac{1}{150}$                       (B)  $\frac{7}{45}$   
(C)  $\frac{7}{30}$                         (D)  $\frac{11}{27}$

6. A coin is dropped into a river from 6 meters above the water's surface. If the coin falls to a depth of 4 meters, what is the total distance that the coin falls?

(Lesson 8-2)

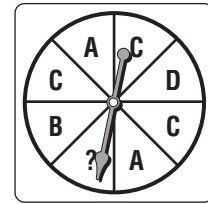
- (F) -6 m                      (G) -4 m  
(H) 4 m                        (I) 10 m

7. Suppose 60% of kids like video games. Predict the number of kids that prefer video games out of a group of 200 kids.

(Lesson 10-8)

- (A) 40                        (B) 60  
(C) 120                      (D) 180

8. What letter should be placed on the spinner so that the probability of landing on that letter would be  $\frac{1}{2}$ ?



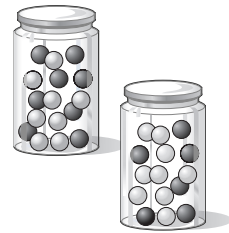
(Lesson 11-1)

- (F) A                      (G) B                      (H) C                      (I) D

9. How many different jeans and shirt combinations can be made with blue jeans and black jeans, and a white shirt, a blue shirt, and a yellow shirt? (Lesson 11-2)

- (A) 6                      (B) 12                      (C) 16                      (D) 20

10. One marble is selected from each jar without looking. Find the probability that both marbles are black. (Lesson 11-5)



- (F)  $\frac{1}{5}$                       (G)  $\frac{6}{15}$                       (H)  $\frac{2}{3}$                       (I)  $\frac{14}{17}$

**TEST-TAKING TIP**

**Question 9** Drawing a tree diagram may help you find the answer.

**PART 2 Short Response/Grid In**

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. Refer to the table below that shows the length of time spent at the zoo.

Family	Time (h)
Martinez	2
Kowalkski	3
Bridi	1
Munemo	4
Smith	2
Elstein	5

What is the mode of the data? (Lesson 2-7)

12. Shaquille is making trail mix. He uses the ingredients below.

Ingredient	Amount (lb)
peanuts	$1\frac{1}{4}$
raisins	$\frac{1}{6}$
dried fruit	$\frac{2}{3}$
almonds	$\frac{1}{4}$

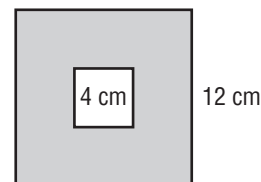
How many pounds of trail mix did he make? (Lesson 6-4)

13. Margaretta notices that 23 out of 31 birds landed in a large oak tree in her backyard and the remaining birds landed in a small pine tree in her backyard. What percent of birds landed in the oak tree? (Lesson 10-7)
14. The probability of a coin landing on heads is  $\frac{1}{2}$ . The 1 in the numerator stands for the number of ways that a coin can land heads up. What does the 2 stand for? (Lesson 11-1)

15. If a dart is thrown at the dartboard shown, what is the probability that it will hit region A? (Lesson 11-4)

B	B	C
C	B	C
A	B	C

16. What is the probability that a dart will land within the large square but outside the small square? (Lesson 11-4)



17. The probability that it will rain on Saturday is 65%. The probability that it will rain on Sunday is 40%. What is the probability that it will rain on both days? (Lesson 11-5)

**PART 3 Extended Response**

Record your answers on a sheet of paper. Show your work.

18. Michelle can select her lunch from the following menu. (Lesson 11-2)

Sandwich	Drink	Fruit
cheese	apple juice	apple
ham	milk	banana
tuna	orange juice	pear

- a. What are the possible combinations of one sandwich, one drink, and one piece of fruit Michelle can choose? Show these combinations in a tree diagram.
- b. If the tuna sandwich were removed from the menu, how many fewer lunch choices would Michelle have?
- c. If an orange is added to the original menu, how many lunch combinations will there be?

